A NEW CHAOTIC ATTRACTOR FROM 2D DISCRETE MAPPING VIA BORDER-COLLISION PERIOD-DOUBLING SCENARIO

ZERAOULIA ELHADJ

Received 12 April 2005

The following map is studied: $(x,y) \to (1+a(|x|-y^2)+y, bx)$. It is proved numerically that this model can display two different chaotic attractors, one is new and the other is a Lozi-type attractor. The new chaotic attractor is allowed via a border-collision period-doubling scenario, which is different from the classical period-doubling bifurcation.

1. Introduction

The discreet mathematical models are gotten directly via scientific experiences, or by the use of the Poincaré section for the study of a continuous model. One of these models is the Henon map. Many papers have described chaotic systems, one of the most famous being a two-dimensional discrete map which models the original Henon map [3, 4, 5, 7, 8]. Moreover, it is possible to change the form of the Henon map for obtaining others chaotic attractors [2, 6], this type of applications is used in secure communications using the notions of chaos.

The Lozi map is 2D noninvertible iterated map proposed by Lozi [6] as follow:

$$(x,y) \longrightarrow (1-a|x|+y, bx). \tag{1.1}$$

This model gives a chaotic attractor called *Lozi attractor* and its shape is resembled to the one shown in Figure 2.1.

2. The proposed model

In this article, we essentially study the following modified Lozi map:

$$(x,y) \longrightarrow \left(1+a\left(|x|-y^2\right)+y, bx\right). \tag{2.1}$$

We show numerically that the new model (2.1) can display two different chaotic attractors; including the classical Lozi-type attractor as shown in Figure 2.1 to Figure 4.2.

Discrete Dynamics in Nature and Society 2005:3 (2005) 235–238
DOI: 10.1155/DDNS.2005.235

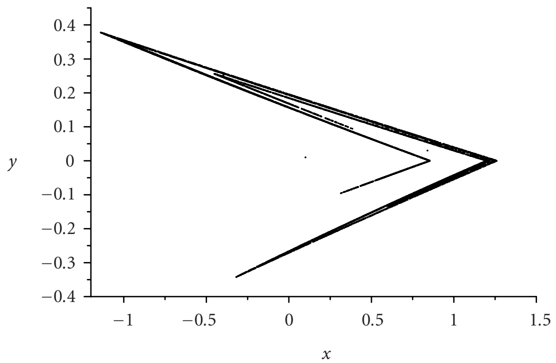

Figure 2.1. A Lozi-type chaotic attractor obtained from system (2.1) for $a = -1.8$, $b = 0.3$.

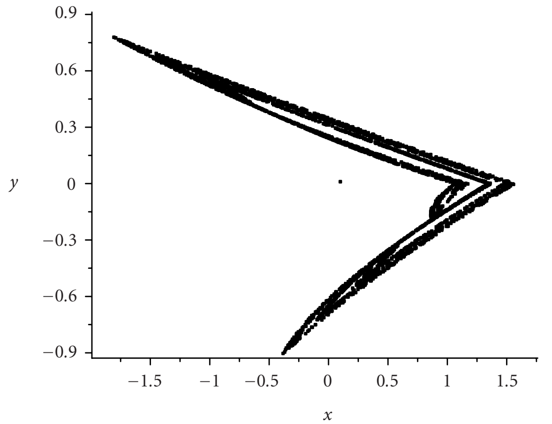

Figure 2.2. A Lozi-type chaotic attractor obtained from system (2.1) for $a = -1.8$, $b = 0.5$.

3. Comparison with the Lozi system

The system (2.1) has the same complexity as the Lozi system (1.1), they are both two-dimensional discrete noninvertible maps. However, the two models are topologically not equivalent because the Lozi system (1.1) is a piecewise-linear map, but model (2.1) is a nonlinear system. In addition; it can be rigorously proved that a non-singular homeomorphism that transforms each system to other does not exist. The proof needed some straightforward but tedious algebra which leads to a system of algebraic equations without solutions.

4. Route to chaos

It is well known that while varying the parameter a, the Hénon attractor is obtained via a period-doubling bifurcation route to chaos as a typical future [3], unless for Lozi map, no

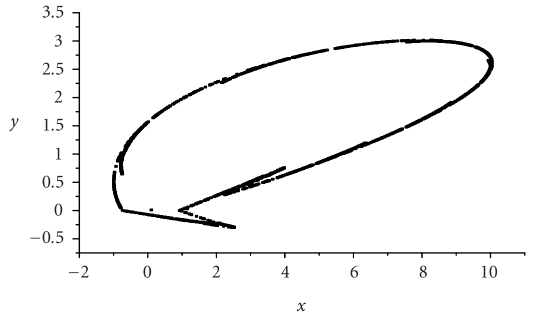

Figure 4.1. A typical orbit of system (2.1) obtained for $a = 1.35$, $b = 0.3$.

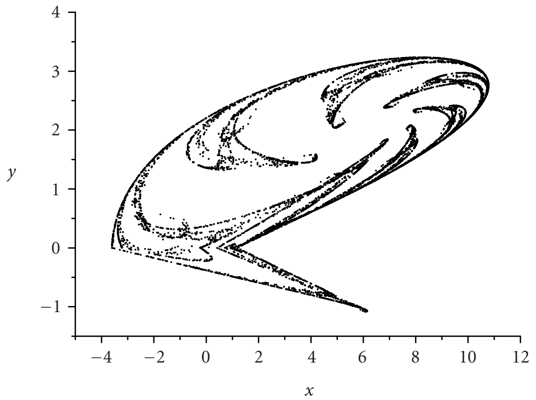

Figure 4.2. The new chaotic attractor obtained for $a = 1.4$, $b = 0.3$.

period doubling route to chaos is allowed, and the attractor goes directly from a border-collision bifurcation developed from a stable periodic orbit [4], and for the same case the new chaotic attractor given by system (2.1) is obtained from a border-collision period-doubling bifurcation scenario [1]; this scenario (see Figure 4.3) is formed by a sequence of pairs of bifurcations, whereby each pair consists of a border-collision bifurcation and a pitchfork bifurcation. Thus, the three chaotic attractors go via different and distinguishable route to chaos as a typical future.

5. Conclusion

This paper reports the finding of a new two-dimensional discrete chaotic attractor obtained via direct modification of the Lozi mapping. The new chaotic attractor is allowed via a border-collision period-doubling scenario. More detailed analysis about the structure, dynamics and bifurcations of the new system (2.1) will be provided in near future.

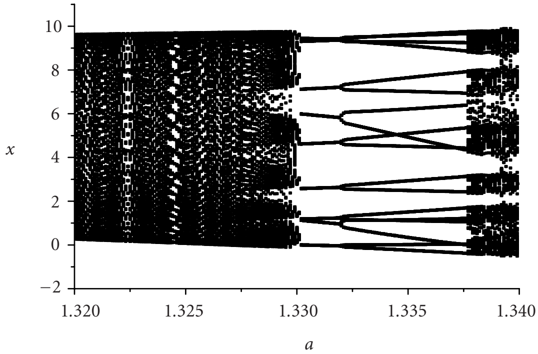

Figure 4.3. Border-collision period-doubling scenario route to chaos observed for system (2.1).

References

[1] V. Avrutin and M. Schanz, *Border-collision period-doubling scenario*, Phys. Rev. E (3) **70** (2004), no. 2, 026222, 11.
[2] M. A. Aziz-Alaoui, C. Robert, and C. Grebogi, *Dynamics of a Hénon-Lozi-type map*, Chaos Solitons Fractals **12** (2001), no. 12, 2323–2341.
[3] M. Benedicks and L. Carleson, *The dynamics of the Hénon map*, Ann. of Math. (2) **133** (1991), no. 1, 73–169.
[4] Y. Cao and Z. Liu, *Strange attractors in the orientation-preserving Lozi map*, Chaos Solitons Fractals **9** (1998), no. 11, 1857–1863.
[5] M. Hénon, *A two-dimensional mapping with a strange attractor*, Comm. Math. Phys. **50** (1976), no. 1, 69–77.
[6] R. Lozi, *Un attracteur étrange du type attracteur de Hénon*, J. Phys. Colloque C5 **39** (1978), Supplément au no. 8, 9–10 (French).
[7] F. R. Marotto, *Chaotic behavior in the Hénon mapping*, Comm. Math. Phys. **68** (1979), no. 2, 187–194.
[8] M. Misiurewicz, *Strange attractors for the Lozi mappings*, Nonlinear Dynamics (Internat. Conf., New York, 1979) (R. G. Heman, ed.), Ann. New York Acad. Sci., vol. 357, New York Academy of Sciences, New York, 1980, pp. 348–358.

Zeraoulia Elhadj: Department of Mathematics, University of Tébéssa, 12000 Tébéssa, Algeria
E-mail address: zelhadj12@yahoo.fr

Mathematical Problems in Engineering

Special Issue on Modeling Experimental Nonlinear Dynamics and Chaotic Scenarios

Call for Papers

Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from "Qualitative Theory of Differential Equations," allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the *Mathematical Problems in Engineering* aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

Manuscript Due	February 1, 2009
First Round of Reviews	May 1, 2009
Publication Date	August 1, 2009

Guest Editors

José Roberto Castilho Piqueira, Telecommunication and Control Engineering Department, Polytechnic School, The University of São Paulo, 05508-970 São Paulo, Brazil; piqueira@lac.usp.br

Elbert E. Neher Macau, Laboratório Associado de Matemática Aplicada e Computação (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), São Josè dos Campos, 12227-010 São Paulo, Brazil ; elbert@lac.inpe.br

Celso Grebogi, Department of Physics, King's College, University of Aberdeen, Aberdeen AB24 3UE, UK; grebogi@abdn.ac.uk