A NEW CHAOTIC ATTRACTOR FROM 2D DISCRETE MAPPING VIA BORDER-COLLISION PERIOD-DOUBLING SCENARIO

ZERAOULIA ELHADJ

Received 12 April 2005

The following map is studied: \((x, y) \rightarrow (1 + a(|x| - y^2) + y, bx)\). It is proved numerically that this model can display two different chaotic attractors, one is new and the other is a Lozi-type attractor. The new chaotic attractor is allowed via a border-collision period-doubling scenario, which is different from the classical period-doubling bifurcation.

1. Introduction

The discreet mathematical models are gotten directly via scientific experiences, or by the use of the Poincaré section for the study of a continuous model. One of these models is the Henon map. Many papers have described chaotic systems, one of the most famous being a two-dimensional discrete map which models the original Henon map \([3, 4, 5, 7, 8]\). Moreover, it is possible to change the form of the Henon map for obtaining others chaotic attractors \([2, 6]\), this type of applications is used in secure communications using the notions of chaos.

The Lozi map is 2D noninvertible iterated map proposed by Lozi \([6]\) as follow:

\[
(x, y) \rightarrow (1 - a|x| + y, bx).
\]

This model gives a chaotic attractor called Lozi attractor and its shape is resembled to the one shown in Figure 2.1.

2. The proposed model

In this article, we essentially study the following modified Lozi map:

\[
(x, y) \rightarrow (1 + a(|x| - y^2) + y, bx).
\]

We show numerically that the new model (2.1) can display two different chaotic attractors; including the classical Lozi-type attractor as shown in Figure 2.1 to Figure 4.2.
3. Comparison with the Lozi system

The system (2.1) has the same complexity as the Lozi system (1.1), they are both two-dimensional discrete noninvertible maps. However, the two models are topologically not equivalent because the Lozi system (1.1) is a piecewise-linear map, but model (2.1) is a nonlinear system. In addition; it can be rigorously proved that a non-singular homeomorphism that transforms each system to other does not exist. The proof needed some straightforward but tedious algebra which leads to a system of algebraic equations without solutions.

4. Route to chaos

It is well known that while varying the parameter $a$, the Hénon attractor is obtained via a period-doubling bifurcation route to chaos as a typical future [3], unless for Lozi map, no
Figure 4.1. A typical orbit of system (2.1) obtained for $a = 1.35$, $b = 0.3$.

Figure 4.2. The new chaotic attractor obtained for $a = 1.4$, $b = 0.3$.

period doubling route to chaos is allowed, and the attractor goes directly from a border-
collision bifurcation developed from a stable periodic orbit [4], and for the same case the
new chaotic attractor given by system (2.1) is obtained from a border-collision period-
doubling bifurcation scenario [1]; this scenario (see Figure 4.3) is formed by a sequence
of pairs of bifurcations, whereby each pair consists of a border-collision bifurcation and a
pitchfork bifurcation. Thus, the three chaotic attractors go via different and distinguish-
able route to chaos as a typical future.

5. Conclusion

This paper reports the finding of a new two-dimensional discrete chaotic attractor ob-
tained via direct modification of the Lozi mapping. The new chaotic attractor is allowed
via a border-collision period-doubling scenario. More detailed analysis about the struc-
ture, dynamics and bifurcations of the new system (2.1) will be provided in near future.
Figure 4.3. Border-collision period-doubling scenario route to chaos observed for system (2.1).

References


Zeraoulia Elhadj: Department of Mathematics, University of Tébessa, 12000 Tébessa, Algeria

E-mail address: zelhadj12@yahoo.fr
Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from “Qualitative Theory of Differential Equations,” allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the *Mathematical Problems in Engineering* aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

<table>
<thead>
<tr>
<th>Manuscript Due</th>
<th>February 1, 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Round of Reviews</td>
<td>May 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>August 1, 2009</td>
</tr>
</tbody>
</table>

**Guest Editors**

José Roberto Castilho Piqueira, Telecommunication and Control Engineering Department, Polytechnic School, The University of São Paulo, 05508-970 São Paulo, Brazil; piqueira@lac.usp.br

Elbert E. Neher Macau, Laboratório Associado de Matemática Aplicada e Computação (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos, 12227-010 São Paulo, Brazil; elbert@lac.inpe.br

Celso Grebogi, Department of Physics, King’s College, University of Aberdeen, Aberdeen AB24 3UE, UK; grebogi@abdn.ac.uk