EMS

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Committee on Mathematics Education

Niveaux de référence pour l'enseignement des mathématiques en Europe

Reference levels in School Mathematics Education in Europe

National Presentation

GERMANY

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§ 1 General description (of the school systems – as it concerns mathematics)

Generally speaking, the most important feature of German „general„ education is the constitutional definition that education is a regional matter (Kulturhoheit der Länder: „Bildung ist Ländersache„), a privilege most attentively maintained and looked after by the regional authorities.

In Germany, general education is seen as a three level project: primary education (usually from grade 1 to 4), lower secondary education („Sekundarstufe 1„, usually from grade 5 to 10) and upper secondary education („Sekundarstufe 2„, especially „Gymnasium„-type from grade 11 to 13). This „horizontal„ stratification is complemented by a vertical differentiation usually beginning in the lower secondary level (grade 5) and offering a whole variety of organisational features depending on regional characteristics (of the so-called „Länder„). The „traditional„ model of lower secondary education was three parallel types of schools (now „Hauptschule„, „Realschule„ and „Gymnasium„ in ascending order of levels of demands), since at least twenty years complemented by the integration of these three traditional schools into comprehensive schools („Gesamtschule„). Some „Länder„ start secondary education in grade 7, in the 16 „Länder„ there are at least half a dozen of different organisational patterns for lower secondary education, sometimes amalgamating for instance „Hauptschule„ and „Realschule„ into a special type of comprehensive school etc. Upper secondary education is marked by the existence of the „Gymnasium„, a type of school/college normally beginning at grade 5 and offering education in one school through to grade 13 (*one* school for the lower *and* upper secondary level). Before primary education, there is also great variability in terms of pre-school education (normally: „Kindergarten„). By law, full-time compulsory education starts at the age of 6 and usually lasts till the age of 16. A short description and official statistics of the German educational system can be found in Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, 1997.

With some minor exceptions, arithmetic (esp. in primary education) or mathematics will be taught in every single grade from grade 1 to 10 (and even through to grade 13 if applicable) with at least 3 „hours„ (of 45 minutes each) a week in lower secondary education. Only in comprehensive schools one would find „inner differentiation„ (i.e. different „tracks„ of mathematics teaching/learning with a spectrum of demand on competencies). Usually one type of school is marked by one (regional!) intended curriculum („Lehrplan„). Without being too unfair, the intended curricula usually can be described as a watering down of a „Gymnasium„-curriculum according to the ideas and aspirations politicians and school administrators have about the type of school they are writing the curriculum for.

Apart from regional specifics, mathematics in lower secondary level will contain something on numbers (in „Gymnasium„ with an explicit discussion of real numbers as the top level), something on algebra (beginning with a simple understanding of formulae up to lengthy inculcation of transposition of algebraic expressions in „Gymnasium„), something on elementary geometry (from the naming of the usual 2D and 3d shapes and related calculations to an analysis of geometric applications beyond congruence and an introduction to proof). Probability and statistics (often called: „stochastics„) are fighting for their acceptance as lower secondary topics in mathematics teaching and learning.

Remark RS:

The description of Germany in EC:DGXXII „Quality and assessment ... - Conclusions Paper„, (EC 1998) is an excellent description, but concentrates on the quarter of the youth going to „Gymnasium„. For the age group of 16, the EC paper is not too informative.

§2 **Main math. objectives** (using BASHMAKOV-structure)

Starting from the three „dimensions„ „mathematical world„, general abilities„ and „application of mathematics„, one has to begin the description of the German situation with the utmost importance of the dimension „mathematical world„ for the intended – and even more so the implemented curriculum. The political debate of the 16 regions („Länder„, see § 1 for the political importance of them) normally concentrates on defining a syllabus which can be followed all over Germany – and this discussion „always„ concentrates on mathematical topics, on the „mathematical world„ and „never„ looks into decisions about „general abilities„ or „applications of mathematics„. Nevertheless, it is common political „prose„ to stress the utmost importance of the applications of mathematics or its „applicability„, the „transfer„ of mathematics from one given situation to a different one and „general abilities„ as well as the importance of (mathematical) language and symbols. Usually, these general claims can be found in introductory paragraphs, but have no consequence in the detailed description of syllabi for the individual regions. Consequently, it is „only„ in syllabi for „Hauptschule„ and/or comprehensive schools that detailed hints and descriptions of modelling activities, mathematical investigations could be found for lower secondary education (grade 5 to 10).

To offer a somehow oversimplifying, but reasonable structure of the intended curriculum at grades 7 to 10, one can identify the following three broad topics: arithmetic/equations and variables/functions as the most important and most time consuming topic of these four grades. Geometry would be the second, but less important mathematical subject (with specific flavour in different types of schools and regions!) while data analysis and statistics fight for a place in the lower secondary syllabi - with a somewhat regionally mixed, at the moment unforeseeable overall result.

This overall picture can be additionally detailed for the lower and upper end of the competency continuum – and because of the regional differences it is difficult to describe the situation in terms of types of schools. As for school algebra (variables, equations and formulae) there will be only a very restricted introduction to variables (if at all) at the lower end of the continuum, solving equations and transposition of formulae tends not be a school subject at this „level„. Some very important formulae (e.g.: the Pythagorean $a^2 + b^2 = c^2$) will be inculcated and learned by heart – maybe even including *all* possible transpositions of the formula (e.g.: area of a rectangle = length x width, hence width = area of rectangle / length). At the upper end of the competency band (say in Gymnasium), this type of procedure is utterly unimaginable.

As for proofs, there will be (nearly) no introduction to proof at the lower end of the spectrum, whereas in Gymnasium in (nearly) every region, explicit mentioning of proof – normally imbedded in geometry lessons around grade 8 - can be found in the syllabi. Even the distinction of premise, statement and proof itself would be found in a Gymansium syllabus for lower secondary level.

As for project oriented work, the situation tends to be the other way round: for the lower „range„ of abilities, curricula tend to name topics integrating different subjects, project work and/or school weeks totally occupied by project work, whereas at the „gymnasium„-level, activities like this seem to be more or less an exception.

This description of the intended curriculum can be easily exemplified by a look into the usual type of textbooks (which have to be approved by the regional ministries of education!). Usually organised along grades (i.e.: one textbook on mathematics for each single class) and specific to every type of lower secondary school (sometimes even specific to a single „Land„), the textbooks use to be structured in terms of mathematical topics relevant to the grade they are written for. Normally, the textbooks offer a way to teach the topics identified in the syllabi (i.e.: give examples, definitions and tasks to work at in the classroom), complemented homework tasks („Aufgaben„) which sometimes fill more than 50% of the pages of a textbook (a German mathematics educator called these pages „plantations of tasks„ („Aufgabenplantagen„), German mathematics education speaks of task oriented didactics –„Aufgabendidaktik„.

As for the use of modern computer technology, there is no national policy on this. Depending on regions and types of school, handheld calculators may be banned or allowed from examinations (including the final examinations at the end of grade 10 for instance for „Realschule„). It seems fair to say that - as a rule - simple handheld calculators tend to be introduced and discussed in grade 6 or 7 and would be then „allowed„ for calculations at school and at home. Computer use (in a professional sense: i.e. PCs or so) do not play a role to be mentioned in a report like this. Neither computer algebra systems („CAS„) nor dynamic geometry software („DGS„) play an important role in the average German classroom – even if innovative teachers and mathematics educators do care for the introduction and use of this technology. With nearly every secondary school now having at least one (!!) computer with access to the Internet (this was a nation-wide campaign last years), the lack of appropriate activities and software becomes more and more apparent.

Apart from mathematical details, the overall philosophy distinguishes the two levels by a simple additum to the „lower„ level: Mathematics has to prepare every student for the everyday life („mathematics for all„), while „gymnasium„ type of schooling additionally prepares for academic studies (not only of mathematics) - hence has to offer some formal mathematics including proof, a certain fluency with basic algebra and equations and knowledge on functions in general including some special functions (like trigonometric functions). Geometry plays a minor role in this overall picture while probability and statistics (in Germany: „stochastics„) is fighting for an inclusion in lower secondary mathematics education.

§ 3 Basic contents

Contrary to the overall educational philosophy and more following the watering down approach usually applied to mathematics teaching, the basic contents in lower German secondary mathematics teaching can be described by giving a table of the usual contents in grades 5 to 10 in Gymnasium - with the watering down effects identified by clearly indicating the topics which tend to be left out for the „lower„ competency range (topics left out in „lower„ levels printed in italics). Before looking into this list of basic contents, it should be kept in mind that this is a description of the intended curriculum (a „synthesis“ of the official syllabi) which also mirrors the „usual“ contents of widely used textbooks for the students. The table does not imply that the contents mentioned is taught to every single student in Gymnasium nor even that a specific topic can be found explicitly in each and every syllabus of Gymnasium of each and every of the 16 regions („Länder“).

For a broad overview in this sense we present the table on the next page.

Table of mathematics content in the German gymnasium (grades 5-10):

(translated from Führer 1997, p. 88; translation R.S. - less frequent topics in brackets)

normally **grade 5-6**	normally **grade 7-8**	normally **grade 9-10**
natural numbers ***N***, arithmetic in ***N*** brackets, alg hierarchy factorisation into prime numbers gcd, scm, rules for divisibility	rule of three proportionality -direct / invers percentage, interest *congruence, congruence mappings, composition of mappings* triangle, quadrilateral, circle, n-	square root *irrational decimals* *HERON-iteration* quadratic equations and functions *parabola*

number systems (Roman, dual) extension to positive rational numbers ***Q*+,** arithmetic in ***Q*+** fractions, decimals plane / spatial basic shapes and figures circumference, area, volume of these shapes drawing of basic shapes and figures conversion and arithmetic of quantities (with dimensions) simple word problems measuring angles symmetry some plane congruencies (reflection, translation - by drawing)	lateral sum of angles and area of triangles, quadrilaterals *perpendicular bisector, height, angle bisector, median of triangles* *certain properties of circles* *construction tasks* perpendicular, parallel *(quadrilaterals inscribed in circles / circumscribing circles)* volume and oblique view of prisms arithmetic with -incl. negative- rational numbers ***Q*** linear equations / *inequalities* *algebraic transformations* *binomial formulae* *relations,* linear functions *coordinates, gradient* *(absolute value)* *2-2 systems of lin. Equations* *(linear optimisation by drawing)* *frequency ,mean* *tree diagram*	*(quadratic inequalities)* *(central) dilation ,similarity* *„Strahlensatz„ (no translation found-RS!)* Pythagoras *and the like* *calculating powers* *power functions* *exponential function* *(logarithm, logarithmic function)* *growth, decay* prism, oblique views circumference, area of circles pyramids, cones, cylinders, spheres *sin, cos, tan* *(sin / cos in triangles)* *expected value, mean variation* *basic calculation of probabilities* *PASCAL's triangle, binomial coefficient* *(BERNOULLI-chain)*

§4 Exemplary topics

4.1 Quadratic equations

Quadratic equations are an exemplary topic which is worth a detailed discussion, because this topic is treated differently for the two levels already within the intended curriculum: Given as a topic for both levels in the usual mathematics syllabi in grade 9 or 10 (see table in §3) it is nevertheless to be treated differently on the two levels: For the level with the lowest aspiration in “Hauptschule, Typ A” in Nordrhein-Westfalen for instance, quadratic equations are mentioned for grade 10 and explicitly restricted to equations with no linear term, to be solved by graphical procedures and checked by using pocket calculators (“auf rein quadratische Gleichungen begrenzen,; Lösungen an der Normalparabel ablesen”, see Nordrhein-Westfalen 1989, p. 65). The “Hauptschule, Typ B” of the same Land (which entitles to further studies in upper secondary schools like Gymnasium and the like) prescribes a more detailed and demanding treatment: it distinguishes both types of quadratic equations (with or without linear term) and prescribes the solution of quadratic equations by means of completing the binomial term or by the formula for the solution(s) of the equation. This dual approach (algebraically and by using the formula) would be the usual diet prescribed for the upper (Gymnasium) level all over Germany – usually treated in grade 9 of Gymnasium. To me, this difference is an excellent illustration of the lowered aspiration in terms of algebra for the lower level of the respective grades.

As a consequence of this treatment of quadratic equations in the respective types of schools, the implemented curriculum (as for instance can be seen in the textbooks) will offer the whole variation from a mere graphical treatment of quadratic equations with no linear term (maybe even restricted to normed equations of the structure $x^2 +a = 0$ with simple numbers for a) to a sophisticated analysis of the number of solutions according to the value of the discriminant even in cases where the equation is given with constants not explicit numbers (like: $ax^2+bx+c=0$). According to the level of analysis there may be also differences in time and depth of related exerceises in solving quadratic equations.

As for the attained curriculum I give the translation of a summary by Andelfinger 1985, p. 221: It is typical for this topic (quadratic equations and functions) that a lot of solutions and calculations (of students) stop short after a first approach to a solution or do not come to an end. This is a consequence of the ... variety and conceptual links (within this topic, all inserts by R.S.) and the delicate question of balancing with the use and usefulness, but also harmful overdose of the well trained algebraic transposition of equations within this field (“Typisch für diesen Themenbereich ist, daß viele Problemlösungen und Rechnungen im Ansatz steckenbleiben oder nicht zum Ende kommen. Dies ist die Folge der ... Vielfalt und Konzeptverbindungen sowie der oft – aber eben nicht immer – erforderlichen Gegensteuerung zur eingeschliffenen Äquivalenzalgebra”, Andelfinger 1985, p. 221). The same source gives a solution frequency below 30% (only of

“Realschule” and “Gymnasium” !!) for equations of the structure $ax^2+bx+c=0$, because students tend not to reach a final solution. My (R.S.) guess would be that knowing the solution formula by heart would be expected from their students by not too small a minority of Gymnasium teachers, while looking up the formula would be a frequent strategy in “Realschule”. The usual “Hauptschul” student will not have met this formula. If a quadratic equation is presented by (x-a)(x-b)=0, solutions are read off by some 40% of the Gymnasium students, a percentage already reduced by an equation of the form (x+a)(x+b)=0.

4.2 Pythagorean theorem

As can be seen from the table in § 3, the Pythogorean theorem is part of the intended curriculum of grade 9-10 (usually: grade 9, in some „Länder“ it may even been taught in grade 8 of Gymnasium). At least the Pythagorean theorem itself („The sum of the areas of the squares of the two legs of a rectangular triangle equals ...“) will be part of the intended and implemented curricula at every „level“ of mathematics education in Germany - and the statement will in reality be told to „every“ student in Germany.

Differences with respect to levels usually occur in the way it is intended and taught: At the „upper“ level (usually: Gymnasium), the related statements about the square of one leg and the rectangle formed by the hypotenuse and the adequate part of it as well as the equality about the square above the height and the appropriate rectangle formed with parts of the hypotenuse („Höhensatz des Euklid“) will be taught - including at least one (of the numerous) proofs of these statements. The inversion of the Pythagorean theorem („if the equation holds, the triangle is a rectangular one“) will also be mentioned (if not proved). This discussion of the statements will be followed by some lessons on „applications“ of the theorem(s) such as conversion of rectangles to squares equal in area (in vice versa), the calculation of lengths of segments and „practical“ problems like the height you can reach with a ladder given the length of the ladder and a certain measure of security for the angle of the ladder.

The „watering down“ for the lower level is done by two important changes: Those heading for a direct way into the workforce normally will not see a mathematical proof of these theorems - and they will usually only learn about the Pythagorean theorem itself - not about the „Höhensatz“ of Euklid nor about the square of one leg of a rectangular triangle. The statement will come out of the blue or „deducted“ from a single example (in most cases: the famous 3-4-5 rectangular triangle). Compared to Gymnasium, there may be even more training on using the Pythogorean theorem for calculating lengths of segments.

From teachers’ experience, it is known that (see Andelfinger 1988, pp. 228-234):

• All students (to a different degree) have problems with a new way of handling the sides of triangles: In contrast to geometry before, the hypotenuse as a side of a rectangular triangle is fundamentally different from the two legs (and they have different, strange names).

• The „application“ of the theorem(s) is difficult because the formula cannot be applied „directly“. In a given drawing, names of segments may be different, if not confusing from the one in the formula learned by heart. In addition to that, the triangle is not presented in the prototypic orientation with the hypotenuse as a horizontal segment.

• It may be even difficult to find a rectangular triangle in a more or less complex configuration. And: the Pythagorean theorem is applied to non-rectangular triangles (as often happen with triangles that have one angle „a bit“ larger than 90° and the longest side in horizontal orientation).

• If we broadly distinguish three levels of competence, there is information on the frequency of solutions to some standard problems:

Geometric construction of square roots and simple conversion of shapes (squares to rectangles etc.) will be solved by 70-75% / 45 % / 35-40%.

The Pythagorean theorem used in word problems or spatial configurations are far more difficult, solution frequencies drop by around 30% (i.e. for the lowest level they are near 0%).

If the solution is not a rational number, this will significantly and negatively affect the solution.

Because of algebraic difficulties, the calculation of a leg given the length of the hypotenuse and the other leg will be successful in 60% / 30% / 10%.

4.3 Similarity

As can be seen from the table in § 3, similarity is part of the intended curriculum *only* for the “upper” level of those heading for academic careers. It is not in the program for every student and will not be explicitly touched for the majority of students till the age 16 / end of compulsory education. Consequently, arguments using similarity are not available to most students when analysing basic (spatial) configurations or shapes.

In Gymnasium, similarity can play two roles: it may either come *before* the Pythagorean theorem and then will serve as an easy way to deduce this group of theorems or it will be part of a learning sequence on geometric transformations which complements and puts in perspective (of preserving length) the congruence transformations. Similarity may also be the aim of a learning sequence on (central) dilation and related geometrical transformations. On the whole, it is important to mention that similarity is *not* a central topic even in Gymnasium programs and may be skipped even in a larger part of German lower secondary Gymnasium classrooms.

As for the attained curriculum on similarity (see Andelfinger 1988, pp. 234-246) one has to mention that similarity is a difficult topic even for “the level-2”-learner in Gymnasium. Andelfinger presents some reason for this: It is an example where an everyday concept gets a new and technical meaning and a complicated, formal way to check or prove it. The formal definition is even somewhat counter-intuitive (“similar” is looking “nearly” equal, i.e. easily detected when parallel sides are give, but very difficult if only the same ratio of lengths or the equality of angles is to be checked). If the figures to be compared are presented in a somehow “perspective” set-up, these will ease the task considerably. Some 60% of the students are convinced that proportion of lengths in similar figures also applies for the respective area and volume.

4.4 Word problems

"Simple word problems" are part of the intended grade 5 curriculum (see table in §3), they may even occur in primary mathematics education. Word problems then will stay with the students till the end of secondary education – and (I, R.S., guess: for a majority of classrooms) may be the only way students are shown "applications" of mathematics. Word problems also form a part of the intended curriculum – as can be shown by looking into textbooks. Usually in grade 7 or 8 – especially in Gymnasium, there is a chapter on how to solve word problems, sometimes even with a sequence of steps to follow for solving word problems like

carefully read the text,

identify the known and unknown in the text,

name the unknown as variable (usually "x")

try to find a graphical, pictorial, iconic representation of the problem,

write down algebraic terms that describe the problem,

write down an equation for the unknown,

solve the equation,

test your solution using the wording of the problem (*not* the equation).

The majority of students do not like word problems exactly because there is no single way to solve them, no algorithm to follow (in contrast to a lot of tasks in mathematics as a school subject).

In Germany, there have been attempts to categorise word problems according to the contexts they present. A widely accepted classification would at least distinguish mathematical (usually arithmetical) word problems, problems from school and everyday life, problems from the physical world (including problems which ask for the place or time where two agents meet when starting and travelling with a given speed) , problems from economics and "whimsical" problems (term introduced by KRYGOWSKA). The large number of problems with percentage and rate of interest come in here too (see next paragraph).

It is difficult to comment on the attained curriculum, last not least because a lot of information from mathematics education is available (see for instance Andelfinger 1985, pp. 234ff), but results from official tests are nearly not accessible. To quote again from one exception is pointing to problem no. 3 of the Bavarian test for Gymnasium (the top level!, for the task and results see appendix 2), where 42,2 % of the possible credits were given to the students' solutions when testing the "top" level of the students in Bavaria in 1998. Tasks no. 8 and 9 in the same test can also be taken as word problems (with task no. 8 asking for a geometrical model whereas task no. 9 heavily draws on the graphical representation of an "empirical" function). With the respective percentages of solutions (no. 8: 17,0 %; no. 9a: 65,9%; no. 9b: 28,6%) one is tempted to point to the fact that in task nos. 3 and 8 the relevant mathematical topic is too long away (belonging to grades 6 and 7) while graphical representations of functions are in constant use in grade 9 (hence the better result in task no. 9a which is directly reading off information from the graph). Task no. 9b is at least a two-step problem with some "real world" interpretation needed in a biological context strange to grade 9 students. "Consequently", the solution percentage drops dramatically.

4.5 Percentage

As can be seen from the table in §3, percentage will usually be in the syllabi for grade 7 and is intended to be taught as an "application" of the rule of three (proportionality). A different solution can be taking percentages as special fractions – and this way of writing the syllabus usually leads to an earlier teaching of percentages (maybe in grade 6). Percentages are normally followed directly by the calculation of rates of interest. So the modelling aspect of the usual rate of interest (it is a societal decision that interest is paid proportional to the amount of money in case and the duration of time) will be obscured by the details of the calculation.

The implemented curriculum on percentages already differs for the two levels to be analysed: Even if percentage can be understood either of a special type of fraction (with 100 as denominator) or as a mere application of proportionality (rule of three), for the lower level 1 percentage would come to the inculcation of three different ways of calculation (or formulae) while for level-2 students ("Gymnasium") textbooks assume that the different tasks related to percentage can be linked or deduced from one another.

As for the attained curriculum, one must not forget that percentages form a major part of word problems – hence look and are difficult to students. The most difficult part of the solution of a percentage task seems not to be the calculation, the arithmetic, but the decision which type of percentage task is disguised in the wording of the problem. So it does not come as a surprise that (only ?) 42,2% of the Bavarian students going to grade 9 of Gymnasium (mind the delay, percentage usually would not have been a topic since about two years) are able to solve the percentage task no. 3 (see appendix 2). As a comment to this result, one has to admit that the calculation of 100% from a reduced price and the rebate offered is one of the more difficult percentage tasks – and the result is a clear reminder that learning is neither irreversible nor simply cumulative.

4.6 Basic (spatial) shapes *(additional topic)*

The main reason why I have chosen this topic can easily be seen from the table in §3: basic spatial shapes and related formulae and calculations are in the intended curriculum for both "reference levels" to be analysed. In Germany, this subject even is one of the few topics given more attention at "level 1" (the non-academic, lower competence level) than at level 2 (those heading for academic studies). The reason behind this can be seen in the importance of the subject for workplace entrance, especially when technical careers are planned. Consequently, this topic usually will not be left out in grade 9 or 10 at the "Hauptschule" or "Realschule" (or comparable schools in some regions which have no such type of school). One is not as sure that the subject will be covered in all Gymnasium classrooms - last not least because some mathematics teachers at Gymnasium may hope to have a better place for this topic in grade 12 or 13.

As for the implemented curriculum, a closer look into textbooks for different types of schools seems to support this description and additionally shows a different way to teach the subject (if it is taught in Gymnasium): If there is time enough a Gymnasium teacher will try to prove the numerous formulae on volume and surfaces of basic shapes like prisms, pyramids, cylinders, cones etc. – and may be even able to point to some common features of the different formulae (by means of an intuitive Cavalieri-principle). For the level-1 student, the topic will mainly made up of a classification of basic shapes, a training in detecting the shapes – even in compound shapes – and then find and apply the appropriate formula (presumably learned by heart or written down before).

The attained curriculum has to cope with special difficulties related to the geometry of space: The topic is rather strange and isolated from the rest of the mathematics curriculum, hence often condensed to only very few lessons (especially in "Gymnasium", see above). As books and papers are still the most important medium in the classroom (and flat by nature), the student has to learn some basic graphical representations of spatial configurations. For spatial configurations, the importance of prototypic images and standard orientations of the graphical representation seems to be even more important than with plane configurations. Most problems have to be stated as word problems (see 4.4) and by this very characteristic are seen as difficult. In addition to this, if calculations are to be done, the number of variables in the formulae is higher than in the rest of lower secondary mathematics education. Andelfinger 1988, p. 251, offers solution percentages of around 30% for these calculations, whereas a spatial task in a Bavarian school mathematics test for grade 9 (only for Gymnasium, with 388 out of 390 schools participating !!) comes up with solution percentages of 16,3% and 15,9% (see appendix 2, task no. 5).

§ 5 Other things

5.1 Regional characteristics

As for regional characteristics, I have to repeat the utmost importance of the regional responsibility for education in Germany ("Bildung ist Ländersache!"). This implies that there are 16 somewhat different educational systems in Germany, with 16 different syllabi for lower secondary education. And lower secondary education even on the overall organisational level is far from uniform in these 16 regions! This makes somewhere around 40 different officially approved syllabi (!!) for lower secondary Mathematics education in Germany.

The most important feature or regional difference may be the decision whether or not the region has a centrally administered final written examination at the end of lower secondary education (normally grade 10, age around 16) or ***not***. My guess is that a minority of students gets its final formal qualification in a centrally set written examination and a local, though loosely centrally controlled additional oral examination (for example in Baden-Württemberg, Bavaria, Thuringia and Saxonia), whereas a majority will get its final examination as locally set for written and oral examinations (with problems from the classroom teacher which was responsible during the last year of schooling).

A political and educational debate on the (dis-)advantages of centrally set examinations already lasts for a couple of years.

5.2 Implementation strategies

The most important means of implementing change (and/or stability) in the curricula of the German regions is a top-down approach using syllabi. With the need of official approval of textbooks and the committees preparing syllabi set up and manned by the (regional!!) ministries of education, this is the “royal path” to change and stability in Mathematics education. The only major exception to this rule was the introduction of “modern Mathematics” in the late 60ies/early 70ies – and this movement “failed” ...

5.3 Teacher training (incl. in-service, non official training)

Pre-service teacher training is reserved to universities (and in some regions: “comparable” institutions like “Pädagogische Hochschulen”) which are closely supervised by regional political authorities which have to approve the plans of study. For secondary education, the teacher of Mathematics usually has at least a three year university study of Mathematics (as one of normally two subjects). The examination (which is of special type and jurisdiction for future teachers, different from the study of a mathematician because of the second subject and introductory lectures to pedagogy/education, under the guidance of the regional authorities) is usually followed by a two-year in-service teacher training completely controlled by regional, political authorities (“Vorbereitungsdienst”). It is only after this (and an additional examination totally independent from university) that state authorities appoint (or not!, also depending on supply and demand) a person as a teacher.

In-service teacher training is usually offered by a mixture of state and “private” suppliers (for instance: churches, political parties, teacher unions, associations of enterprises; for “new” technology/computers also: suppliers of hard-/software). In-service teacher training in Germany tends to be a rather erratic, somewhat chaotic enterprise.

5.4 Resources available to teachers

Most regions have at least one centre which offers information and training to its teachers – but this usually cannot offer the training on demand by the schools. In Germany, there is a wide range of journals on mathematics education and teaching of varying quality, targeted population and number of sold copies. Nevertheless, it is common to start from the assumption that less than 10 % of the (Mathematics) teachers ever read information on mathematics education apart from the textbook they use in their classrooms.

5.5 Problems already detected

Instead of offering a list of problems some 10 pages long, I want to identify a problem pertinent not only to mathematics education and teaching: With the regional responsibility for education in

Germany, every idea or proposal can usually be commented by hinting to at least one region that already follows this proposal and does no better or worse. With no national authority to decide and implement a new proposal, it is very difficult to come to a national diagnosis of a problem and/or achieve a change on a national scale – even if it is an incremental one.

5.6 Data of general / local results

With the regional organisation of secondary education and the differences between the individual regions, it is very difficult to general results on Mathematics teaching in Germany. In some sense, the TIMSS-results (together with FIMS and SIMS) were the first national evaluation of the German mathematics education. As for regional results, some regions offer the problems (not the results) of their central examination (see the URLs in the appendix 1) or special activities following TIMSS (e.g. Bavaria, see appendix 2). Even when directly asking for this results at the regional ministry of education and identifying oneself as a university professor searching for scientific information, it is difficult or impossible to get detailed (regional) results.

5.7 Examples of inspiring activities

Following the (bad or deceiving or normal) results of the TIMS-study, regional authorities in a joint effort (!!) agreed to have a nation-wide program to develop mathematics and science education. They agreed of 11 dimensions of this program (see Baumert et al. 1997). Following from the regional responsibility, the details of this program have been decided locally seem to vary to a considerable degree.

Somehow strange to the German system and specific to Mathematics as the single subject of the enterprise, there is an association of more than 400 mathematics teachers who elaborate and exchange lesson plans on subjects agreed upon in advance (email-adress: mued.ev@t-online.de). MUED offers a documentation of lessons for secondary mathematics teaching which seems to be singular in Austria, Germany and Switzerland.

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Appendix 1: Some URL on regional examinations

For „Realschule“ in Baden-Württemberg:

http://www.bw.schule.de/realschule/rsonline/pruef.htm

For various types of schools in Thüringen:

http://www.thueringen.de/tkm/hauptseiten/medien1.htm

Appendix 2: From a Bavarian Mathematics test in grade 9 of Gymnasium

The complete test is available as an „exe.file“ from Besancon. Here I only give the results for the test-items an the two tasks mentioned in this report.

Task no. 3:

Aufgabe 3

Petra hat sich ein neues Fahrrad gekauft. „Ich habe 30% Rabatt bekommen und nur 635 DM und ein paar Pfennige bezahlt“, verkündet sie daheim voller Stolz.
Ihre Mutter überschlägt im Kopf: „Dann kostet das Rad ja regulär ungefähr ...“

Markiere die Angabe, die am nächsten am ursprünglichen Preis liegt.

A) 840 DM

B) 860 DM

C) 890 DM

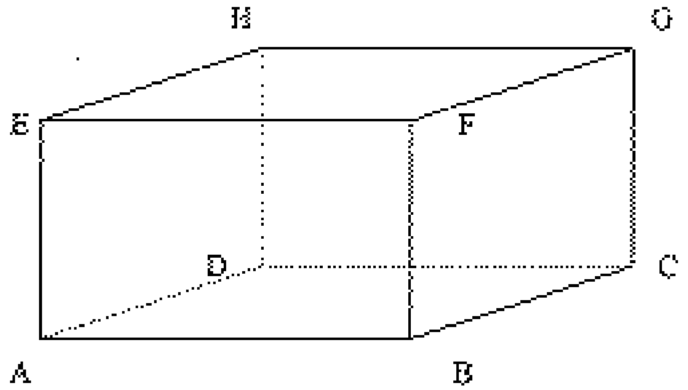
D) 910 DM

E) 820 DM

Task no. 5:

Aufgabe 5

Durch die Punkte A,... ,H ist ein Quader festgelegt.



a) Wir betrachten Geraden, die durch den Punkt F und eine andere Ecke des Quaders gehen. Welche dieser Geraden schneiden die Gerade EC?

....

b) Beschreibe die Menge aller Punkte, welche die durch E, C und G festgelegte Ebene mit der durch A, B und C festgelegten Ebene gemeinsam hat.

....

results of the test: