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Committee on Mathematics Education

National Presentation

ITALY

(by Lucia Grugnetti and Vinicio Villani)

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1.GENERAL DESCRIPTION OF THE ITALIAN SCHOOL SYSTEM

1.1. Structure

The Italian educational system is strongly centralized. All structural decisions are made by the Ministry of Public Education which sends directives to the regional and provincial educational authorities (“Dirigenti Scolastici Regionali” and “Provveditori agli Studi”), which in turn interact with the schools of their respective geographic regions.

Compulsory education begins at age six. Up until the year 2000, pupils were required to attend five years of primary school, and three years of middle school. Thus compulsory education finished at age fourteen with an exam called Licenza Media (Middle school diploma), organized at the local level, by individual schools.

After this exam, students who intended to continue their studies could choose among various types of high schools (in Italy called “Senior secondary schools”), each in turn divided into numerous tracks:

- Lycaeum (Classical and scientific high schools)
- Technical high schools (split into various tracks)
- Vocational high schools (split into many tracks).

In most cases, these high schools lasted, and still last, five years and concluded at the end of grade 13 with a national final exam (called “Esame di maturità” or, more officially, “State Exam”).

So far we have described the situation of the past, but beginning with the year 2001 a reform is about to be undertaken, which is expected to modify the structure of the Italian educational system in depth.

More precisely:

(a) Compulsory education increases to age fifteen (already in effect since the current school year).

(b) The pre-university education will be reduced to twelve years, that is, in the future, it will go from age six to eighteen (this change will require some time to go into effect).

(c) Middle school will be abolished and thus in the future all pre-university education will be divided into only two levels: primary school, lasting seven years, and secondary school, lasting five years (one predicts that this new division will not completely be in effect until the year 2007).

(d) For those students who leave the educational system at the end of compulsory education (i.e. after seven years of primary school, and two years of secondary school) additional part-time vocational training will be provided.

Many practical aspects of this reform are still unclear, beginning with the drafting of syllabi of schools undergoing restructuring.

Until now admission to university faculties has been open to all students who passed the final high school exam (except for a few courses with restricted admission). As a certain percentage of students repeat a class during their pre-university schooling, the average age of university registration is closer to twenty years than to nineteen.

1.2. Specific aspects of the Italian school system

Middle school syllabi presently in effect, were issued in 1979.

Mathematics and Science subjects (Elements of Chemistry, Physics and Natural Sciences) are joined, and taught by the same teacher.

On average, three periods in the week, out of 30, are dedicated to mathematics.

As far as high schools are concerned, even though traditional syllabi, over half a century old, still officially exist, they have in fact been replaced by experimental syllabi, which have unsystematically overlapped in recent years.

The most widespread experimental curriculum, and the one to which we will be referring in the following section, is the so called "Brocca Syllabus" (from the name of the chairman of the commission which elaborated it about ten years ago).

Until now differentiation of studies occurs after middle school, i.e. after grade 8. At this time students must choose the type of school in which they want to continue their studies. Various majors are possible within the same type of school. In the experimental Brocca Syllabus, for mathematics, two different types of programs are offered during the first two years of high school (i.e. for students aged 14 to 16). The first is called "weak mathematics" (on average three periods per week, in a weekly schedule of 30 periods total), and the second is called "strong mathematics" (on average four or five periods per week, in a weekly schedule of 30 periods).

Regarding the planned reform, schools will enjoy a higher degree of autonomy (which has been quite limited until now), and they will also have more leeway with regard to the implementation of national teaching syllabi. For the moment it is too soon to foresee what this will imply in concrete terms.

Among the most specific aspects of the Italian educational system it is worthwhile to cite three:

* One teacher accompanies the same group of pupils for several years (three in middle school, between two and five in high school).

* Middle school and high school syllabi are arranged thematically, and not year by year. In contrast to the traditions in other countries, these syllabi are rather concise (for example seven printed pages are dedicated to mathematics for the three years of middle school, and sixteen printed pages are dedicated to mathematics in the first two years of high school).

* During the first two years of high school, mathematics and the basics of computer science are taught by the same teacher. It is unfortunate that, although computer lab is managed by the mathematics teacher, computational subjects remain separated from those dealt with in the periods dedicated to traditional mathematics. Only a minority of teachers make an effort to link these two aspects (for example by using educational software such as Cabri or Derive).

2. MAIN MATHEMATICAL OBJECTIVES

According to the experimental Brocca syllabus, in the first two years of high school instruction in mathematics (and computer science) should aim to promote:

1) the development of intuitive and logical skills;
2) the ability to use heuristic procedures;
3) the maturing of abstract and conceptual thinking;
4) the ability to reason inductively and deductively;
5) the habit of precision in speech;
6) the ability to reason consistently and in a well-argued manner;
7) the awareness of cultural and technological aspects emerging from newcomputer devices;
8) an interest in the historical impact of some important events in the development of mathematical thinking
9)

3. MAIN TOPICS FOR THE FIRST TWO YEARS OF HIGH SCHOOL (AGE 14 - 16)

The list of topics presented in the Brocca syllabi for the first two years in classes adopting "strong mathematics" is the following:

Theme 1. PLANE AND SPATIAL GEOMETRY

- Euclidean plane and space and its isometric transformations. Geometric figures and their properties. Equidecomposable polygons; Pythagorean theorem.

- Dilatations and similarities. Thales theorem (i.e.: In a parallel projection between two lines, the lenghts of corresponding segments are proportional).

- Cartesian plane: straight line, parabola, equilateral hyperbola.

- Cosine and sine of convex angles. Relations between sides and angles in right angled triangles.

- Significant examples of geometrical transformations in the space. Symmetries in simple geometric solids.

Theme 2. NUMERICAL SETS AND COMPUTATION

- Algebraic and order structures and their properties for natural, integer and rational numbers.

- Approximate values and their use in elementary computation. Intuitive introduction to real numbers. Square roots and simple operations on them.

- Algebraic language and computation with letters: monomials, polynomials, algebraic fractions.

- Equations and systems of first and second degree. Linear inequalities.

Theme 3. RELATIONS AND FUNCTIONS

- Sets and operations on them. Basics of combinatorics.

- Composition laws for particular structures. Cartesian product. Binary relations: order and equivalence. Applications (functions).

- Functions x _ax + b, x _ax2 + bx + c, x _a/x and their graphs.

Theme 4. NOTIONS OF PROBABILITY AND STATISTICS

- Elementary probability spaces: random events, disjoint events and the "rule of sum".

- Conditional probability, composite probability. Independent events and the"rule of product".

- Notions of descriptive statistics: data acquisition, synthesis values, variability indexes.

Theme 5. NOTIONS OF LOGIC AND COMPUTER SCIENCES

- Propositional logic: elementary propositions and connectives, truth value of a composite proposition .

- Logical inference, main rules of deduction

- Variables, predicates, quantifiers.

- Analysis, organization and representation of data, structured construction of algorithms and their representation.

- Finite automata, alphabets, words and generative grammars. Syntax and semantics. First introduction to formal languages.

A COMPUTER LAB is also planned.

4. EXEMPLARY TOPICS

4.1. Quadratic Equations

Point 4 of Theme 2 of the Brocca syllabi (for “strong mathematics” only) reads: Equations and systems of first and second degree. Linear inequalities.

Usually these are dealt with in the second year of high school (16 year old students); sometimes at the end of the first year.

Until a few years ago they were introduced through formal algebraic computations (typically the completion of the square). At present, in certain textbooks at least, there is the tendency to start with the geometrical study of parabolas, topic listed by the Brocca syllabi as point 3 of Theme 1; in this context the solutions of second degree equations are considered in terms of zeros of the functions which represent the parabolas.

The way the topic is approached is almost always internal to mathematics, without significant links to physics or situations connected to problems dealt with in other disciplines or to the real world.

4.2. Pythagorean Theorem

Point 1 of Theme 1 of the Brocca syllabi reads in both the “strong” and the “weak” versions: Euclidean plane and its isometric transformations. Geometric figures and their properties. Equidecomposable polygons, Pythagorean theorem.

Generally students will have already encountered the Pythagorean theorem in the second year of middle school (pupils between ages 12 and 13). It is one of the few theorems which are “proved” at this educational level (usually by considering the visual evidence of suitable geometrical constructions).

We present here an example of how a middle school textbook deals with this topic - generally for the second year (Francesco, Speranza, La Matematica, Parole Cose Numeri Figure, Zanichelli, 1984, p. 242).

13. Il teorema di Pitagora

Pitagora fu un matematico e filosofo greco: nacque a Samo e visse a Crotone intorno al 500 a.C. Fondò una «scuola», cioè un gruppo di studiosi. A questa scuola risalgono molte scoperte.

Rammenta che un triangolo rettangolo è un triangolo con un angolo retto. I lati che formano l'angolo retto si dicono *cateti*, l'altro lato si dice *ipotenusa*.

Se conosciamo le lunghezze dei cateti, possiamo cominciare a disegnare il triangolo rettangolo: si vede che anche la lunghezza dell'ipotenusa è determinata (fig. 36). Quanto sia non è facile dirlo: puoi provare a disegnare un po' di triangoli rettangoli (meglio se i cateti hanno, in centimetri, misure intere), e scrivere le tre misure: dei cateti e dell'ipotenusa.

Quanto sia lunga l'ipotenusa, lo dice il famoso teorema di Pitagora:

proprietà

La somma delle aree dei quadrati costruiti sui cateti di un triangolo rettangolo è uguale all'area del quadrato costruito sull'ipotenusa.

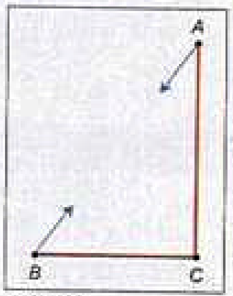


Figura 36

Chiamiamo ABC il triangolo rettangolo, con l'angolo retto in C (fig. 37). Diciamo b la lunghezza del cateto AC, a quella del cateto BC, c quella dell'ipotenusa. Prolunghiamo CB di un segmento di lunghezza b, fino al punto D. Costruiamo il quadrato $CDEF$, di lato CD. Prendiamo i punti G e H in modo che la distanza di G da D e quella di H da E sia a. La distanza di G da E è ...; quella di H da F è ... .

I punti A, B, G, H sono i vertici d'un quadrato. Infatti i lati hanno tutti lunghezza uguale (nei triangoli ABC, BDG si ha... quindi ...; nei triangoli ...). Gli angoli sono tutti retti (ad esempio, $\widehat{FAH}$ e $\widehat{CAB}$ hanno per somma un angolo retto; $\widehat{BAH}$ si ottiene togliendo questi due angoli da un angolo piatto, quindi è retto).

L'area del quadrato interno è c^2, quella di $CDEF$ è $(a+b)^2$. La differenza è data dall'area di quattro triangoli rettangoli, ciascuno di area $\frac{1}{2}ab$. Tutti e quattro hanno area $4 \cdot \frac{1}{2}ab = 2ab$. Quindi

$$c^2 + 2ab = (a+b)^2 = a^2 + 2ab + b^2.$$

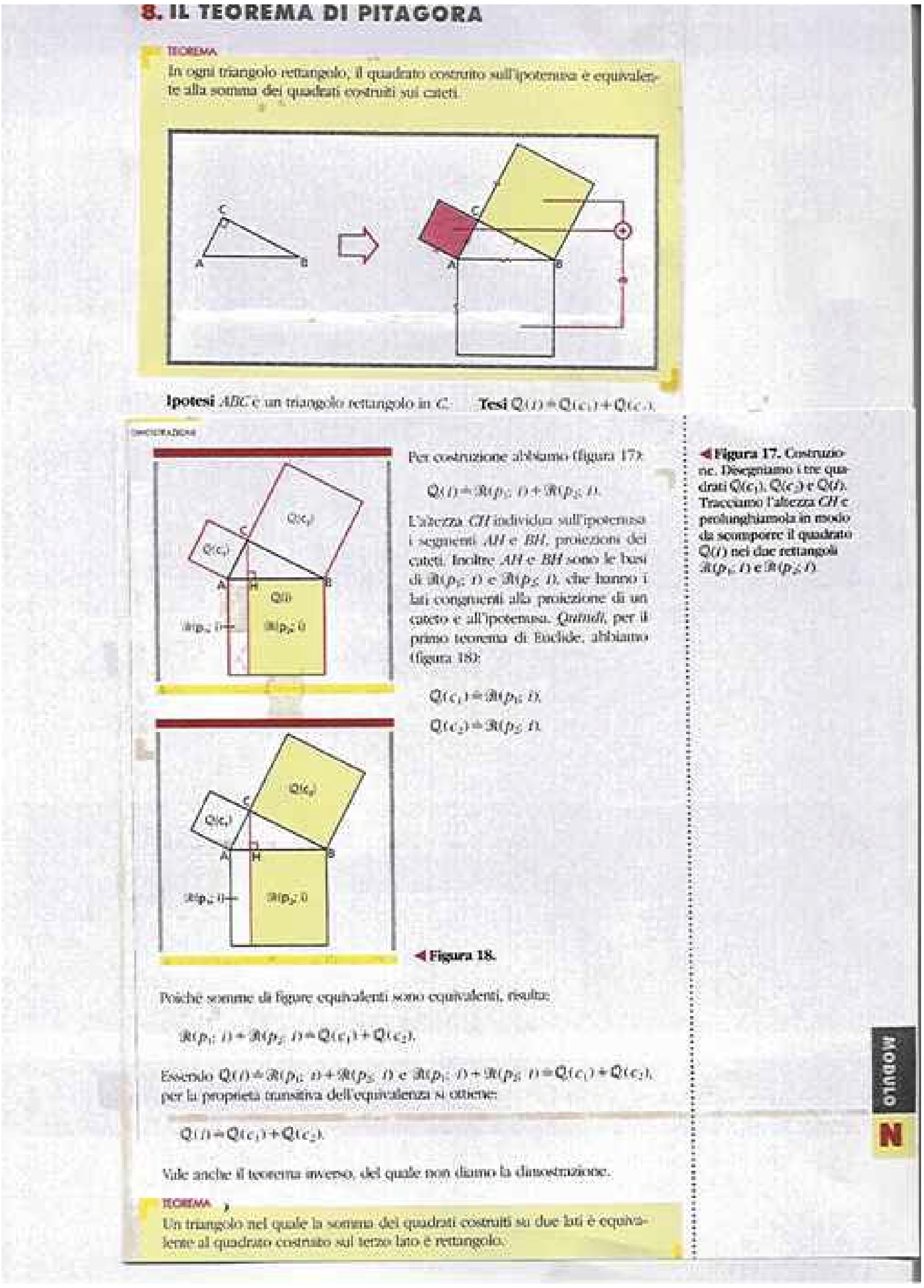
Sottraiamo $2ab$ da entrambe le parti, e abbiamo

$$c^2 = a^2 + b^2.$$

Se conosco le misure a e b dei cateti, quella dell'ipotenusa è $\sqrt{a^2 + b^2}$.

This same theorem is then taken up again at the beginning of high school (in the first or second year) by framing the Pythagorean theorem in a more systematic construction of Euclidean geometry.

We present here an example of how a high school textbook for the first two years deals with this topic (Anna Trifone, Massimo Bergamini, Matematica per moduli 2, Zanichelli, 1998, pp 14 - 15 of mod. N).



8. IL TEOREMA DI PITAGORA

TEOREMA

In ogni triangolo rettangolo, il quadrato costruito sull'ipotenusa è equivalente alla somma dei quadrati costruiti sui cateti.

Ipotesi ABC è un triangolo rettangolo in C.   **Tesi** $Q(i) \doteq Q(c_1) + Q(c_2)$.

DIMOSTRAZIONE

Per costruzione abbiamo (figura 17):

$Q(i) \doteq \mathcal{R}(p_1; i) + \mathcal{R}(p_2; i)$.

L'altezza CH individua sull'ipotenusa i segmenti AH e BH, proiezioni dei cateti. Inoltre AH e BH sono le basi di $\mathcal{R}(p_1; i)$ e $\mathcal{R}(p_2; i)$, che hanno i lati congruenti alla proiezione di un cateto e all'ipotenusa. *Quindi*, per il primo teorema di Euclide, abbiamo (figura 18):

$Q(c_1) \doteq \mathcal{R}(p_1; i)$,

$Q(c_2) \doteq \mathcal{R}(p_2; i)$.

Figura 17. Costruzione. Disegniamo i tre quadrati $Q(c_1)$, $Q(c_2)$ e $Q(i)$. Tracciamo l'altezza CH e prolunghiamola in modo da scomporre il quadrato $Q(i)$ nei due rettangoli $\mathcal{R}(p_1; i)$ e $\mathcal{R}(p_2; i)$.

Figura 18.

Poiché somme di figure equivalenti sono equivalenti, risulta:

$\mathcal{R}(p_1; i) + \mathcal{R}(p_2; i) \doteq Q(c_1) + Q(c_2)$.

Essendo $Q(i) \doteq \mathcal{R}(p_1; i) + \mathcal{R}(p_2; i)$ e $\mathcal{R}(p_1; i) + \mathcal{R}(p_2; i) \doteq Q(c_1) + Q(c_2)$, per la proprietà transitiva dell'equivalenza si ottiene:

$Q(i) \doteq Q(c_1) + Q(c_2)$.

Vale anche il teorema inverso, del quale non diamo la dimostrazione.

TEOREMA

Un triangolo nel quale la somma dei quadrati costruiti su due lati è equivalente al quadrato costruito sul terzo lato è rettangolo.

MODULO N

Here are two fairly typical examples involving the Pythagorean theorem.

Middle school: "In a right angled isosceles triangle one cathetus measures 26 cm. Calculate the perimeter and the area of the triangle."

High school: "Prove that the square built on the height of an equilateral triangle is equivalent to three times the square built on half of one side."

4.3. Similarity

In the Brocca syllabi this topic appears (for "strong mathematics" only) as point 2 in Theme 1: Dilatations and similarities. Thales theorem.

It is usually dealt with in the second year (students aged 16).

But, also in this case, a few simple considerations on similar figures are already dealt with in the third year of middle school. Similarity, seen as a global transformation of two (or three) dimensional space, was absent in the traditional syllabi. It is instead present in the Brocca syllabi, but in current educational practice one continues to emphasize, rather than similarity, only criteria stating sufficient conditions in order that particular geometric figures (triangles and polygons) be similar.

4.4. Word Problems

Already in elementary school pupils are confronted with word problems.

Here is how official syllabi present them :

Primary school (1985 syllabi).

Mathematical thinking is characterized by problem solving activities. This is in accord with the tendency of the child to ask questions and look for answers. As a consequence, basic mathematical notions have to be founded and constructed starting from concrete problem situations which derive from the everyday experiences of the child. In addition, they should offer the chance to ascertain the child's previous mathematical attainment, which tools and solving strategies he uses, and what difficulties he might encounter.

It is however important to avoid proceeding in a scattered and disorderly way; instead one must strive towards a progressive organization of knowledge.

Middle school (1979 syllabi).

It is important to keep in mind that "solving a problem" does not only mean applying fixed rules to situations which have already been schematized, but it also means facing problems in their raw state, thereby requiring the pupil to fully take charge of the translation in mathematical terms.

Brocca syllabi (first two years of high school):

One can not be under the illusion that it is possible to start with mathematics as a ready-made discipline, i. e. with theories and concepts already elaborated and codified, without paying attention to the constructive procedures which concern them. It is instead important to begin with didactic situations which favor the emergence of problems that can be dealt with mathematically, the practice of heuristic procedures to solve them, the genesis of concepts and of theories, the approach to axiomatic and formal systems. The natural sources of these situations are the real world, mathematics itself, and all the other sciences. All this leads to possible interdisciplinary teaching opportunities, first in the discovery and characterization of the specific fields and methods of the different disciplines, and afterwards in linking them togerther in a conceptual synthesis (...) The choice of situations and problems is part of a larger didactic project, which one realizes through the evaluation of the psychological readiness and the starting level of the students, the analysis and the setting of learning objectives, the analysis and selection of contents, the choice of methodologies and suitable techniques, the adoption of adequate assessment procedures.

However, one must note that the attention dedicated to these types of problems decreases in high school, where the practice of teaching is more traditional than in elementary and middle school.

EXAMPLES OF PROBLEMS:

Primary school:

"I have to pay 700 Lire. I have one 200 Lire coin, two 100 Lire coins, and several 50 Lire coins. In how many ways can I pay my bill?"

Middle school: "A farmer bought 48 meters of wire to make a fence; how can the rectangular fence which he is going to make be? What is the greatest area the fenced in zone can have? Try to draw in scale a few of these fences."

High school: "We want to plant 21 tulip bulbs in a rectangular plot. We want to have them in equal rows, with the condition that the number of bulbs in each row exceeds by four the number of the rows.

How many rows of bulbs do we need to plant?"

4.5. Percentage

This topic is not explicitly mentioned in the high school syllabi. It is usually dealt with in the second and third year of middle school, and only occasionally retrieved in the following years. As a consequence, illiteracy with respect to this topic exists, even in the high school classes which follow the "strong" mathematical syllabus.

5. OTHER ASPECTS

A typical aspect of the Italian school tradition is the importance given to oral testing (not only in mathematics). Just as an example, at the final examination of the classical Lyceum, mathematics appears only as an oral subject, without any written test (even if, in point of fact, oral testing may include the solution of written exercises, at the blackboard). The strong point of well conducted oral testing is the interaction which takes place between teacher and students, as well as the acquisition, on the students side, of the habit of expressing and arguing, skills that are more difficult to acquire if evaluation of the achievement is based only on written testing, or worse, only on multiple choice testing.

On the other hand, a weak point is the excessive time needed and the difficulty the teacher has in objectively evaluating the results of oral testing.

5.1. Regional variations

Considering that syllabi are national, one would not expect relevant variation within the same types of schools. Instead, the level of learning (in mathematics as in other disciplines) varies greatly due to reasons related also to socio - cultural differences.

5.2 Implementation Strategies

Specific national structures for implementation do not exist. Mainly, this is left to the personal initiative of individual teachers or to activities organized by non-government associations.

5.3. Teacher training

Until now teachers have been trained at universities, with four year courses focussed on the study of a specific discipline (e.g. Mathematics) and ending with a university degree called “Laurea”. Recruitment was based on state

exams always within the specific discipline. Didactic and pedagogical preparation was absent.

In some cases even preparation within the specific discipline was inadequate (for example, middle school teachers who are mainly graduates in biology, have a weak foundation in mathematics). Beginning with the year 2000, two year post-graduate courses designed especially to prepare future teachers have been established.

5.4. Resources available to teachers

At the national, regional, and local level didactic resource centers exist which, however, have very little impact on the majority of teachers. Several journals devoted to mathematics education for the secondary level exist. It can be estimated that they reach less than 10% of the teaching body.

5.5. Problems already detected and improvements foreseen

* Italian textbooks are extremely redundant and scattered. In most cases students' textbooks are not accompanied by teacher's guides.

* In most schools mathematics instruction is traditional (theoretical presentation by the teacher, routine exercises, written and oral testing in class). One hopes that the advent of new computer technologies and the foreseen increased autonomy of schools will lead in the future to a better teaching style.

5.6. Data on general/local results

Reliable, official information is lacking.

5.7. Examples of inspiring activities

Given the absence of national structures for updating and innovation, small groups of particularly motivated teachers organize, on a voluntary basis, seminars on specific themes (for example: computer technology and its didactic impact, training courses on topics recently introduced in the syllabi, such as probability and statistics, or transformation geometry, etc.) and which are lead by university teachers and researchers of disciplines in the field of mathematical education.

These initiatives, of a decidedly high level, involve however a very low percentage of the global teaching body and thus have a very small impact on the entire national educational system.

Another aspect which deserves mention is the recent progressive expansion of mathematics competitions for students of all educational levels. This could contribute to increase the interest of young people for mathematics and to improve the public opinion's image of the discipline.

6. ADDITIONAL TOPIC: PROBABILITY AND STATISTICS

The introduction of this topic in Italian schools is quite recent (it was first introduced in the 1979 middle school syllabi, then in the 1985 elementary school syllabi, and finally in the experimental high school syllabi of 1986).

The syllabi are rather advanced, but come up against many teachers insufficiently prepared, and consequently reluctant to give the subject any in-depth treatment.

The theoretical aspects (which include critical reflection and a comparison between the various possible layouts of probability) prevail over applicative aspects and the presentation of "realistic" situations susceptible to being analyzed from a statistical-probabilistic point of view.

The official syllabi with regard to this topic assert*:*

Primary school (1985 syllabi)

Concepts, principles and skills connected with the statistical representation of facts, phenomena, and processes, as well as with the elaboration of judgements and predictions in situations of uncertainty are of considerable importance in education.

The introduction of the first elements of probability, which could take place at the end of elementary school, aims to prepare in the child an intuitive foundation on which it would be possible at a later stage to develop rational analysis in situations of uncertainty.

The classical definition of probability - as the ratio between the number of favorable cases and the number of possible cases in symmetrical random situations - can not be assumed as a starting point, but is rather the point of arrival of a well graduated activity.

In the development of this itinerary the construction and analysis of procedures and of algorithms - numerical and non numerical - can be realized also with an initial but coherent and productive use of suitable tools for computing and elaborating information.

Middle school (1979 syllabi)

Theme: "Mathematics of certainty and mathematics of chance"

a) True / false statements and probabilistic statements. Correct use of logical connectives (and, or, not): their interpretation as operators on sets and their application to electrical circuits.

b) Statistical measurements and their graphic representation (histograms, areograms ...); frequency; averages.

c) Random events; elements of probability and their applications.

Brocca syllabi (first two years of high school):

Theme 4: " Elements of probability and statistics"

On the one hand, the study of probability develops a correct approach to the analysis of situations in conditions of uncertainty, supplying tools to rationally deal with the information available and to make coherent decisions, and, on the other hand, it provides new domains in which it is possible to handle interesting mathematized examples.

For the consolidation of a probabilistic mentality, which could orient the student also in judgements made in daily life, a well-shaped introduction to the various definitions of probability, and rich examples taken from real situations are essential.

The study of probability further creates a context in which formalization and abstraction can lead students to an axiomatic structuring of the theory. In problem solving it is good to use multiple tools such as combinatory calculus, the Euler-Venn diagrams, and various types of graphs.

The contents of the statistics section are the occasion to fine tune in a more rigorous and formalized way concepts and tools in part already known, by suggesting a deeper familiarization through applications to interdisciplinary problems and contexts. Of particular importance are the analysis and interpretation of data, presented in various forms: charts, graphs or more synthetic indices, in order to enable the student to correctly and critically make use of the statistical information he may encounter.

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