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Reference levels in School Mathematics Education in Europe

National Presentation

ENGLAND

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[NOTE: The current English National Curriculum in Mathematics [1] has recently been reviewed. The changes are awaiting government approval. The draft proposals [2] (which are quite detailed) are rather different in form from the current curriculum (which is very vague). My information is that the final version will be very close to the draft proposals, so any reference to "the recently revised" version of the curriculum are based on the draft proposals [2], even though these have not yet been approved. Any necessary modifications to this report will be made later.]

1. General description of the school system (from the point of view of mathematics)

1.1 History

Until 1988 education in England was a local responsibility - with control vested in around 100 "Local Education Authorities" (LEAs). Each LEA received the bulk of its funding from central government; LEAs also raised a percentage of their budget through local taxes. They were responsible for compulsory education (ages 5-16) and for further education (ages 16-19 and adult vocational training). The LEA provided advice and support to schools and colleges. However, the details of what was taught and how it was taught remained the responsibility of the individual school: there were no national or local curricula.

This apparent recipe for anarchy was constrained by the fact that

(a) what was taught at secondary level was strongly influenced by "public" examinations at age 16 and age 18, and

(b) until the early 1970s, what was taught at primary level was influenced by selection tests for entry to "grammar schools" at age 10/11. (During the late 1960s and 1970s selection at age 10/11 was largely abandoned, and most of the existing grammar schools were reconstituted as "comprehensive" (all ability) schools. Any such change had to be decided at a local level, and so had electoral implications for local politicians; hence some LEAs adopted compromise schemes with the result that we still have 166 selective secondary schools - out of 4000 or so - in England. However, the present government has introduced proposals which are intended to make it easier for LEAs to change the status of the remaining grammar schools.)

At neither primary nor secondary level was there any tradition of devising detailed curricula or teaching plans: some schools devised their own teaching plans, but most mathematics departments selected and followed a scheme or textbook series which, in their judgement, matched the abilities of their pupils and which prepared them for the next exam they faced. There was (and is) no official procedure for vetting or "approving" textbooks: schools are not only free to choose, but are given no official information to assist in that choice.

The examinations which effectively determined the curriculum were not designed or controlled by government agencies.

The entry tests for grammar schools were administered by LEAs - often using commercially available tests in English, Mathematics and Reasoning. The "public" examinations were provided by a large number of "examination boards" - most of which had historical connections with groups of universities. The main exam boards originally existed to provide "matriculation tests" for university entrance, but in the 1960s they began to coordinate locally based exams at age 16 for non-academic pupils.

The move away from grammar schools coincided with an increasing concern to provide for the (previously neglected) "bottom 75%".

The consequences of these changes were often driven as much by local politics and administrative concerns as by educational policy. For example:

(a) The potential market for the examination boards more than doubled; and the desire to generate income from the larger part of this market was sometimes stronger than the demand for educational principles to guide these new developments.

(b) Grammar schools had contained the bulk of highly qualified teachers, and so were the natural focus for academic educational provision at age 16-19. Abandoning grammar schools created a vacuum at senior high school level. In the political euphoria following "comprehensivisation" almost all schools were allowed to offer academic courses for pupils aged 16-19 - although it was clear from the outset that this was unworkable.

To have allowed a minority of schools to retain academic provision for age 16-19 would have looked like reinventing grammar schools by the back door. So many LEAs moved towards a new pattern of provision - with secondary schools restricted to ages 11-16, while separate "colleges" provided academic and vocational courses for those aged 16-19.

This led to an outflow of qualified mathematics teachers from 11-16 schools into 16-19 colleges. (Offical statistics [3] show that in 1992 in maintained (i.e. publicly funded) secondary schools

(i) only one third of the mathematics lessons for pupils aged 11-14 were taught by teachers whose pre-service training included a degree with mathematics as a main component;

(ii) while almost two thirds of mathematics lessons for pupils aged 16-18 were taught by teachers with such a qualification.)

1.2 The present

Like the British constitution, many English institutions reflect what outsiders must see as an incoherent muddle. However, given a measure of stability (and, in the case of education, a sufficiently large, critical mass of competent teachers), this incoherence has often proved historically valuable by leaving room for innovation, flexibility and compromise.

The dramatic changes in England from 1965 to 1985 made educational change essential; but they also undermined social and political stability. The administration introduced a new style of "radical politics" in which change was imposed through a peaceful version of "permanent revolution" - an endless string of initiatives and "reforms" which achieved central control by undermining the traditional stability of English life. (This style of administration has since been adopted much more widely.)

One of the main goals of the (right wing) government of the 1980s was to restrict the power of (mostly left wing) urban LEAs. To this end they exploited the language of "accountability", and used this to justify mechanisms for central control. In education, this led naturally to the idea of a centrally controlled national curriculum, with centrally administered national assessment. As indicated above, some change was needed; but the motivation for what happened was more political and bureaucratic than educational.

England has no tradition of central control in education. Unlike certain other European countries, we had no cadre of officials who understood the conditions under which central control in education can work, who appreciated its limitations, or who knew how to administer a centrally controlled system.

The changes that were eventually made in the late 1980s were strongly influenced by the position of *mathematics* in the school curriculum. In response to widespread complaints about the mathematical abilities of school leavers, the government established in 1978 an inquiry - which came to be known as the "Cockcroft Committee" (after its Chair, Sir Wilfred Cockcroft, a professor of mathematics). This committee was instructed "to consider the teaching of mathematics ... with particular regard to the mathematics required in further and higher education, employment and adult life generally". They interpreted this instruction in a strongly utilitarian way: the very limited mathematics which adults actually use was accepted as indicating what really needed to be taught at school level. This had profound consequences. In particular, though the report itself was mostly moderate in tone, it was used by others to persuade a whole generation of English educators and administrators to undervalue the "mathematical world", and to overvalue "generic skills" (such as teamwork and problem-solving), "technology", and "applications" with little mathematical content.

This was the era of plausible slogans. Technology, we were told, had changed completely what schoolchildren needed to learn. They no longer needed to learn facts or algorithms, but should rather be taught "generic skills" such as "problem-solving" or "how to learn".

Multiplication tables and written algorithms were too narrow; instead children needed a "sense of number". Noone explained how one could achieve such a "number sense" without mastering traditional arithmetic, or how one could "solve problems" without the requisite facts and algorithms. Having woken up to the fact that "the mathematical world" on its own is not *sufficient* to ensure that school leavers can use their mathematics, English mathematics educators appeared to conclude that this formal universe was also *unnecessary* - and even unhelpful. Teachers were encouraged to believe that one could teach pupils to use mathematics directly, without first grappling with the awkward abstractions of traditional school mathematics.

The Cockcroft committee reported in 1982 [4]. Its analysis and recommendations were concerned solely with mathematics; but they stemmed from tensions and conflicting demands which affected all subjects. In particular, the report had to address the kind of changes which might provide a better deal for "the bottom 75%".

The Cockcroft Report made some important and valuable recommendations, and achieved a remarkable balance between "progressive" and "traditional" views of mathematics education (even if those who subsequently used the report to further their own agendas were often less sensitive to this need for balance).

However, the report had two main weaknesses:

(i) It was too concerned to resolve tensions within the mathematics education community, and so failed to notice more telling administrative and political trends (such as the increasing calls for "accountability" and for central control).

(ii) The report was strongly influenced by emerging evidence of the gulf between the "intended" and the "achieved" curriculum. Unfortunately it failed to distinguish between universal facts and local pathology. For example, data formt eh late 1970s which showed that certain topics appeared to be "too hard" for most English 14 year olds was interpreted at face value without looking across the Channel to see whether this was a universal fact or a temporary local aberration. That is, the report did not take account of what was routinely achieved in other European countries.

In the years immediately following the Cockcroft Report, successive Ministers (with orders from the then Prime Minister, Margaret Thatcher) grappled with the problem of how to regain control of an educational system which had lost its traditional contraints - but with limited success.

Some partial improvements were implemented. For example, until 1985 public examinations at age 16 were offered on two levels:

(a) the traditional "O level" exams for the top 25-30%, and

(b) an unsatisfactory watered down version of these exams for the next 30-35%. Both exams officially excluded the bottom 40%.In line with the purely mathematical recommendations of the Cockcroft Report, 1985 saw the introduction of a uniform system of examinations at age 16 for all subjects.

The "system" was uniform, but there were dozens of different exams offered by different agencies. However, they all had to observe certain common criteria - which led to the introduction of vetting procedures which marked the beginning of central control!

The new system of exams served a larger fraction of the age group, and the syllabus for weaker students was designed specifically to cater for their needs and abilities. (However, since this requirement was interpreted in line with the comments in (ii) above - the resulting syllabuses were partly responsible for institutionalising low expectations.) The detailed history of this period remains to be written. However, it would appear that modest changes of this kind were insufficient for the then Prime Minister. Hence a new Minister of Education was appointed to introduce wholesale change.

The Education Reform Act (1988) :

(i) introduced a compulsory national curriculum in ten subjects; and

(ii) provided for the subsequent introduction of centrally controlled national testing at ages 7, 11, 14 and 16.

The administration underestimated the difficulties of what was being attempted. To start the process of drafting and implementing an agreed curriculum (in ten subjects, with no experience) it was thought obvious to start with Mathematics.

The situation was made worse by two things.

(i) The abolition of the grammar schools had created a national taboo: it was no longer possible to discuss the idea of "selection", or to consider devising different curricula for identifiably different groups before the end of compulsory education at age 16. Hence the National Curriculum had to appear to provide equal opportunities for all.

(ii) However, the need to make some provision for different abilities had been highlighted by the Cockcroft report, which had revealed the dramatic gulf between the achievments of the best and the worst English pupils at age 11 - a phenomenon which was summarised in the phrase "the seven year gap" (meaning that, after removing outliers, the best 11 year olds were as good as the average 14 year old, while the weakest were no better than the average 7 year old).

Instead of proposing measures to reduce this "gap" (as in most countries), it was assumed to be a fact of life. Thus we rejected the idea of a common "year-by-year" curriculum for all pupils up to an age where divergence made it natural for pupils to attend different kinds of schools - with each type of school having a slightly different curriculum. Rather we tried to devise the maximal curriculum for ages 5-16 in the form of a single ladder - with ten rungs, up which all pupils would climb *at their own pace*. Schools were expected to accommodate those who progressed very slowly (perhaps reaching only the third or fourth rung by the age of 16) as well as those who could reach the top much more quickly (perhaps by the age of 13). This created an unmanageable diversity within each classroom.

A problem had been recognised, but the curriculum structure which was adopted reflected a refusal to tackle it centrally: instead the buck was passed to individual teachers and schools.

Worse, this "bottom-up" model of the curriculum as a single ladder meant that the curriculum specification of important topics was determined by the needs and limitations of the slowest pupils. Thus, the details of what was needed at the level of introductory algebra were specified in the same way for those who might proceed to study science and engineering as for those who would struggle to understand the simplest formula.

The resulting mathematics curriculum which was forced through in 1989 was unworkable. Its mathematical and educational shortcomings were denied until relatively recently (and are still not acknowledged openly); but one could not escape the consequences of the contradictions inherent in the structure of this curriculum. However, these consequences could not be effectively addressed as long as their origin was denied. Thus there were repeated attempts (in 1991, in 1993 and in 1995) to revise the mathematics curriculum - but only "to make it easier to administer", not to correct its genuine weaknesses [1].

Notwithstanding the inherent flaws in the curriculum, by the mid 1990s teachers had got used to it and were tired of change. Politicians and bureaucrats used this weariness to argue that no further changes were needed.

The subsequent very public struggle - fuelled in late 1995 by the London Mathematical Society report "Tackling the mathematics problem" [5] and by the very poor TIMSS (Population 2) results [6] - led eventually to the most recent review [2], which has just been completed (and which is to be implemented from September 2000). Officially this review was only allowed to make minor changes in content. Hence most of the changes made had to be presented as attempts to clarify the intention of the existing (rather vague) curriculum by specifying content in a more detailed and structured way, while still trying to encourage desirable teaching styles.

In parallel with, and perhaps more important than, this recent curriculum review has been the National Numeracy Strategy [7,8]. This scheme constitutes a radical shift in the way mathematics will be taught at primary level. The approach has been piloted over the last 2-3 years and will be implemented in all primary schools from September 1999. In contrast to the vague and contradictory guidance of the last 15 years, the Numeracy Strategy provides a highly structured model for the teaching of elementary mathematics. The approach is pragmatic, has been formulated in great haste, and has many shortcomings; but is generally accepted as a serious - and on the whole sensible - attempt to improve the achievement of "the bottom 75%" of 11 year old primary school leavers. The stategy offers schools a very detailed year-by-year curriculum, and incorporates a requirement that each primary school class should devote 45-60 minutes each morning to mathematics.

Though the strategy has many good features - its emphasis on "numeracy" rather than "mathematics", and the absence of any similar program for secondary schools, is worrying. Also, the year-by-year structure of the Numeracy Strategy is logically incompatible with the "level-by-level" ladder structure of the National Curriculum; rather than confront this issue and make a rational choice, English pragmatism appears content to embrace both - for the time being.

2. Main mathematical objectives (using Bashmakov structure)

The lack of any explicit guiding principles in the English National Curriculum makes it difficult to identify the "main mathematical objectives". The following remarks indicate the relative emphasis on "the mathematical world", "applications" and "general abilities".

The 1989, 1991, 1993 and 1995 versions of the curriculum all played down the role of "the mathematical world". The motivation was understandable, but superficial.

England has no tradition of pedagogy and didactics. There is therefore no accepted formal way of analysing the challenges which confront the mathematics teacher, or of communicating intended modifications to existing or intending teachers. The only vehicles are therefore pragmatic ones: from textbooks, syllabuses and examinations, to personal example and encouragment to "reflect on one's experience" (though without a theoretical framework).

This tradition proved unable to handle the shift in the 1970s from

(a) a "top-down", university driven agenda to

(b) "education for all".

The new majority were taught and examined in (a watered down version of) the old minority tradition, with unsatisfactory results. There was also increasing evidence that even the very best pupils undestood much less than had been assumed. This led to a reaction against "formal methods". This reaction was strengthened by those who (ignoring the lessons of the last 400 years) claimed that traditional disciplines, each with its separate range of techniques and methods, were now less important than "generic skills" (or general abilities), utility (or applications), and technology. Instead of reassessing the kind of "formal mathematics" which had to be taught, and the way it should be taught, to serve the new majority better, there sprang up a belief that informal methods might suffice. The National Curriculum (and the associated assessment) encouraged teachers:

(1) To see school mathematics as being motivated and justified by its uses ("We believe it should be a fundamental principle that no topic should be included unless it can be developed sufficiently for it to be applied in a way which pupils can understand" [4, p.133]. "Pupils should be given opportunities to use and apply mathematics in practical tasks [and] in real-life problems" [1. p.11]). (

2) To concentrate on encouraging pupils to "use what they know" with confidence, rather than to try to use more formal methods which they do not understand. ("very many pupils in secondary schools are at present being required to follow mathematics syllabuses whose content is too great and which are not suited to their level of attainment" [4. p.132]).

(3) To ensure that pupils are given opportunities to explore, and to investigate, simple situations with some mathematical content, to "find ways of overcoming difficulties that arise, develop and use their own strategies" [1. p.11].

In this spirit, the 1995 curriculum began - not with the three "content strands" (Number and algebra, Shape and space, Handling data), but with a strand called "Using and applying mathematics", which emphasised "making and monitoring decisions to solve problems, communicating mathematically, and developing skills of mathematical reasoning" [1].

While some teachers managed to use this emphasis to improve their teaching, the approach suffered from a failure to analyse the relationship between :

(i) achieving fluency in mathematical techniques, and

(ii) being confident in the use of basic techniques.

The most recent review of the curriculum [2] has made a nominal attempt to incorporate "using and applying" as part of each content strand. Previous versions of the curriculum emphasised "general abilities" and "applications", while failing to specify clearly either which aspects of "the mathematical world" were most important, or how they should be taught in order to derive the most benefit from pupils' willingness to "use what they know" with confidence. The most recent revision of the curriculum spells out in much greater detail those aspects of "the mathematical world" that are felt to be important, and encourages teachers to bring "using and applying" back into the mainstream, but gives little indication of how this should be done.

3. Basic contents

The [draft] revised version of the mathematics curriculum [2] covers 46 densely printed A4 pages.

The content is presented under three main headings :

- Number and algebra
- Shape, space and measures
- Handling data.

Since the curriculum is still specified in terms of "levels" rather than "grades" it is difficult to infer from what is written what is expected of the average pupil, or of the majority of pupils.

"Number and algebra" means essentially "Number" until age 11, after which algebra begins to play an increasing role. The curriculum for ages 11-16 contains most of what one would expect if the top 25% or so are to solve quadratic equations, to handle simple quadratic functions graphically, and perhaps to find the points of intersection of a straight line and a circle. However, the curriculum for the bottom 70% or so suggests that they should not be expected to work with quadratic expressions.

The title "Shape, space and measures" indicates something of the tension which lies scarcely beneath the surface. The reluctance to use the word "geometry" in the title reflects the earlier distaste for any hint of formal methods. The presentation of content is made more difficult by the need to accommodate the persistent belief that "transformations" offer a viable approach to "useful geometry for the majority". Nevertheless, the strand includes most of the basic material one would expect under the heading "elementary geometry" (geometry of lines and triangles; approximate and exact constructions; congruence; Pythagoras; and similarity).

The treatment varies from informal (for the majority) to semi-formal (for the top 25% or so - who also meet the basic circle theorems).

"Space" plays a limited role (exploring 3D shapes, working with cuboids and prisms, with the top 25% required to use Pythagoras in 3D).

"Measures" are included in this strand only because there seemed to be nowhere else to put them.

"Handling data" focusses on "collecting data, processing and representing data, interpreting and discussing results", while also trying to convey the unpredictable nature of random processes, the kind of questions that can be addressed using statistical methods, and sources of bias in statistical data. There is an obligation on teachers to involve pupils in designing experiments and surveys so that they have to decide what data to collect in order to answer a simple question.

There are no official indications of the relative weighting of the three strands (except that Handling data does not appear explicitly during the first 2-3 years, but is viewed at that level as a natural part of "Number"). In the early years (age 5-7) "Number" may occupy 80% of the available time, with "Shape, space and measures" occupying 20%. In the later years (age 12-15) "Number and algebra" may occupy 55% of the time, "Shape, space and measures" 30%, and "Handling data" 15%.

4. Exemplary topics

4.1 Quadratic equations

For the majority of pupils there is no mention of any of the ingredients needed for solving quadratic equations (factorising simple quadratics, quadratic functions and their graphical representation, surds, etc.) - either in the current Naitonal Curriculum [1] or in the revised curriculum [2].

The current version of the National Curriculum [1] does not recognise the central role of quadratic equations, of factorisation, of completing the square, or of the quadratic formula - even for the top 25%. Instead it suggests vaguely that, by the age of 16 such pupils :

"[should] sketch and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations", and

"should be taught to solve a range of ... quadratic and higher order polynomial equations, selecting the most appropriate method for the problem concerned, including trial-and-improvement methods."

These phrases are open to different interpetations. However, the failure to highlight the central position of quadratics, and the fact that the only solution method mentioned is that of "trial-and-improvement" (i.e. intelligent guesswork) encouraged the interpretation that little more is expected than that pupils "solve" quadratics having small (probably positive) integer roots.

The reference to "graphs that model real situations" - and the absence of any mention of factorisation, or completing the square - illustrates the extent to which concern for "applications" had undermined the balance between this and "the mathematical world".

One of the accepted weaknesses of the current curriculum [1] which led to the recent review was the realisation that elementary algebra needed to be more tightly specified. For the top 25% or so of pupils the 1999 revision proposes:

(i) "use surds in exact calculations"

(ii) "solve quadratic equations by factorisation, by completing the square, and by using the 'quadratic formula'"

(iii) "solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown and the other linear in one unkown and quadratic in the other, or where the second is of the form x^2 + y^2 = r^2"

(iv) " plot graphs of quadratic functions ... find the intersection points of the graphs of a linear and a quadratic function and knowing that these are approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions".

However, the bottom 70% meet only linear expressions, and so are not expected to cover any of these topics.

4.2 Pythagorean theorem

In the 1995 version of the National Curriculum [1] the top 45% or so (only) of 16 year olds are expected to "understand, recall and use Pythagoras' theorem ... when solving problems in two dimensions". The only additional reference to Pythagoras' theorem is the comment that the top 10% or so should "use ... Pythagoras' theorem ... when solving problems in two or three dimensions". Nowhere is there any suggestion that pupils should see a *proof* - the clear implication being that this is thought to be unnecessary.

The 1999 revision of the curriculum [2] leaves the requirements for the majority of pupils unchanged, but goes into more detail for the top 25% or so, stating:

"Pupils should be taught to:

understand, recall and use Pythagoras' theorem in 2D, progressing then to 3D problems;

investigate the geometry of cuboids including cubes, and of shapes made from cuboids, including the use of Pythagoras theorem to calculate lengths in 3D;

There is still no mention of the significance of, or the need to present, a proof.

4.3 Similarity

In the 1995 version of the National Curriculum [1] similarity remains exclusively informal for all but a tiny minority of pupils. The average 16 year old was expected to "enlarge shapes by a positive whole-number scale factor", but the bottom 30-40% are not even required to do this much.

The top 25% or so were required only to "understand and use mathematical similarity". There was no explanation of what this means. There is no link with trigonometry. Nor is the conection with enlargements clarified.

The recent revised curriculum [2] leaves the low expectations for the bottom 75% unchanged. However, it goes into more detail for the top 25% or so. Thus, by the age of 16, such pupils should be taught to

- "understand similarity of triangles and of other plane figures", and use this to make geometric inferences".
- "understand that enlargments are specified by a centre and a (positive) scale factor; recognise, visualise and construct enlargements of given objects; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not; then use positive and negative scale factors"
- "recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; understand the implication of enlargement for perimeter ; use and interpret maps and scale drawings; understand the difference between formulae for perimeter, area and volume by considering dimensions; understand and use the effect of enlargement on areas and volumes of shapes and solids"

There is no mention of the link between similarity and euclidean geometry (Midpoint Theorem, or Thales Theorem).

4.4 Word problems

The previous version of the National Curriculum [1] encouraged informal methods to the exclusion of key formal methods. In such a setting, most problems tended to be presented using words (even when they would sometimes be better presented using symbols): for example, pupils are to be taught to :

"express simple functions initially in words and then symbolically"

"find and describe in words the rule for the next term, or nth term of a sequence where the rule is linear".

However, there was no mention of traditional "word problems".

Pupils were expected to solve simple equations. But, despite the emphasis on "using and applying", there was no indication that "word problems" have a central role to play in linking the familiar world of language and everyday experience with the world of numbers and symbols. (This is consistent with the observation that many mathematics educators in English-speaking countries see "word problems" as part of an outmoded tradition which they would like to replace.)

The 1999 revised curriculum [2] is scarcely any better in this regard - though we do find the explicit requirement (for the top 25% only) that

"pupils should be taught to solve problems and word problems."

When the inclusion of such an item was proposed it became clear that most of those responsible for administering the curriculum did not understand what was meant by the expression "word problems". There would appear to be little interest in, or understanding of, the contribution which a "didactics of word problems" can make to pupils' appreciation of elementary algebra, or to their feeling for the process whereby mathematical symbols can be used to model and to solve real problems.

4.5 Percentage

In the 1995 version of the National Curriculum [1] percentage is presented simply as a technique to be mastered. It is mentioned repeatedly, but with no real mathematical context.

"Pupils should be taught to: calculate with negative numbers, decimals, fractions, percentages and ratio; ... understand when and how to use fractions and percentages to make proportional comparisons".

The vast majority of pupils were expected to "use simple percentages", or to "calculate percentage parts of quantities, using a calculator where appropriate". The average pupil was expected to be "aware of which number to consider as 100 per cent, or a whole, in problems involving comparisons, and use this to evaluate one number as a fraction or percentage of another; [and to] understand and use the equivalences between fractions decimals and percentages." The latter expectation was expected only of a small minority of pupils.

There was no explicit mention of problems involving percentage increase and decrease, or of compound interest; other indicators show that these were expected only of a relatively small minority.

The revised curriculum [2] attempts to clarify the multiplicative/operator character of percentage, and so offers a slightly better foundation for linking percentage with fractions and ratio. The average pupil is supposed to be taught to:

"understand that percentage means 'number of parts per hundred'; interpret percentage as the operator 'so many hundredths of' ; use percentage in real life situations"

" convert simple fractions of a whole to percentages of the whole and vice versa; then understand the multiplicative nature of percentages as operators"

"solve simple percentage problems, including increase and decrease".

In addition, the top 25% are expected to

"solve reverse percentage [problems]"

"[perform written calculations involving] repeated proportional change using a multiplier raised to a power (e.g. compound interest)".

4.6 Additional theme : Proof

In the 1995 version of the National Curriculum [1], the word proof occurs just once. Though words like "reasoning" and "justification" appear, these have come to be seen as injunctions to encourage pupils to formulate "their own explanations": there is no indication that such explanations be mathematically correct - or that, if they are incorrect, then they should be corrected. Indeed, except for the world of number, there is no framework within which one could assess the correctness or otherwise of most such explanations. (Even in the realm of algebra, pupils have been officially encouraged to guess (using unsubstantiated "generalisation" from observed numerical patterns)rather than to calculate.)

The recent revision represents a first attempt to fill this vacuum

(i) by embedding exploratory and investigational work within the content strands, and

(ii) by presenting each content strand in a way which emphasises connections, without imposing an excessive or premature logical structure.

The 1999 "geometry" curriculum explicitly includes local deductive chains (involving lines, parallels, and angle; ruler and compass constructions; triangles and congruence; and the short sequence of circle theorems for the top 25%) though a fierce rearguard action is still being fought to try to exclude this material.).

However, there is still little indication of why it is important for pupils to move beyond informal experience to an abstract semi-formal "mathematical world", or of the difference between experimental observation and proof. Neither is there a professional consensus concerning the need to clarify such distinctions in order to help more pupils appreciate the nature of elementary mathematics, its strengths and limitations, and its role in modern society.

This lack of understanding is perhaps best illustrated by the persistent refusal of those responsible for the revised curriculum to recognise that Pythagoras' theorem is so unexpected a result that most countries would take it for granted that one should hestitate before separating the result from its proof .

5. Other things

5.1 Regional variations

In recent years the curriculum in Wales has been closely tied to that in England - except for the fact that the Welsh language is an additional compulsory subject up to the age of 16: Welsh is the language of instruction in almost 40% of primary schools and perhaps 15-20% of secondary schools.

In the recent curriculum review, Wales did not feel the same pressures to change the mathematics curriculum, so has made very few changes to the version [1] which England has found so unsatisfactory.

The system in Northern Ireland is more loosely linked to that in England.

Scotland has its own rather different education system.

5.2 Implementation strategies

The Qualifications and Curriculum Authority (QCA) has responsibility for the curriculum. They approve examination syllabuses for age 16 and 18 and administer national tests at age 7, 11 and 14. They also publish supporting materials for teachers and distribute them to schools. However there are no other permanent support structures to help teachers implement the curriculum, or to address any problems that arise. (QCA does organise small working groups from time to time to consider specific problems.)

A new initiative will sometimes have a short term budget which allows them to offer one-off courses with a specific short-term objective. For example, the National Numeracy Strategy has appointed fifteen regional directors operating training schemes for primary teachers, and has recently provided three days of training for three members of staff from each of the 16 000 primary schools in the country to help schools introduce the strategy effectively in September 1999. A similar training program for secondary schools will take place during the summer term 2001.

5.3 Teacher training (including in-service and non-official training)

Intitial (i.e. pre-service) teacher training is the responsibility of the Teacher Training Agency (TTA), which distributes funds to universities, to colleges and to groups of schools to train teachers. Pre-service training of secondary mathematics teachers usually takes one of three forms (though there are variations):

(a) 4 year B.Ed. - mostly for primary teachers, but with some secondary provision;

(b) 1 year post-graduate certificate, taken by students who have already graduated with a specialist degree including sufficient mathematics;

(c) school-based training :

We now have a National Curriculum for teacher training, which specifies what trainees should know before they are granted "Qualified Teacher Status". Though this has some good features, the official requirements cannot be achieved with the current trainees in the time available, and there is no support mechanism, which might allow trainees to continue to be supervised during their first years of teaching.

This, together with the drastic shortage of applicants and difficulties in the funding regime, indicates that there are serious problems in the pre-service training of mathematics teachers in England.

In-service training (INSET) used to be provided by LEAs, but they have been largely deprived of this function. Responsibility for INSET was then given to the TTA, but this responsibility was recently removed, and currently rests with the Department for Education and Employment. However, they do not appear to have any coherent policy or mechanism for delivering INSET - other than for specific targeted programmes, such as that associated with the Numeracy Strategy. Thus t in-service support is often now provided on an ad hoc basis by private organisations. While this leaves schools free to seek out the support they need, the new system makes it unlikely that either schools or providers will try to address anything other than the most pressing short-term issues.

5.4 Resources available to teachers

Each school has a "staff development" budget, and a set of perceived priorities. Teachers (or departments) may apply to use part of this money to attend teachers' conferences or in-service days which fit in with the school's priorities.

The Department for Education and Employment funds a small number of short courses; these tend to address particular issues (such as the use of technology in teaching mathematics) and the number of places is severely limited.

LEAs used to encourage experienced teachers to take secondment for further study, but this ceased some time ago.

5.5 Problems already detected and improvements planned

TIMSS highlighted serious weaknesses at primary and lower secondary level. These have been addressed partly through the Numeracy Strategy [7,8,9] and the 1999 revision of the National Curriculum [2]. The recent TIMSS-R results indicate the extent of the problem to be tackled. The effectiveness of recent changes at secondary level will be seriously affected by the serious shortage of well qualified teachers.

One problem which is being slowly recognised is that of the "de-professionalisation of teachers". The pressures which have been exerted on schools in recent years to try to "change the culture" have undermined the sense of professional autonomy which is an essential ingredient in all good teaching: teachers feel that their every move is being monitored, often using inappropriate criteria.

5.6 Data of general/local results

Test data (for ages 7, 11, 14 - with some base-line tests at age 5) has been increasingly used to exert pressures on schools. The results (for ages 11 and 14, and the results of public exams at age 16 and 18) are tabulated and published in national "league tables". This practice is supposed to exert pressure on unsatisfactory schools, and achieves this to some extent, But the real effect is rather different: since the results are so public, schools feel obliged to concentrate their efforts on achieving the sort of results which look best in the published tables. Concerns for pupils' "education" is thereby subordinated to a strategy which tries to "beat the system" - even though the quality of the tests is generally poor, and the method of presenting the results is simplistic.

Every school is supposed to be inspected once in every 5-6 years. The inspectors' written report assesses the quality of the teaching and the effectivenes of the management. The report is published and is available to the public. In addition to the written report, the inspectors make a verbal report to the senior management in the school. Special measures are adopted to "rescue" schools which are deemed to be "failing" or in difficulties - with variable success..

5.7 Examples of inspiring activities

The current educational climate, with increasing scrutiny of schools' performance, demands for accountability, and central control make it difficult for those who wish to engage in "inspiring activities". Publishers are increasingly publishing only those texts which promise to improve schools' performance on those tests whose results are published, and teachers are under increasing pressure to focus on narrow, test-centred goals. There are nevertheless several areas which deserve mention under this heading.

(a) Teachers' cooperatives : Many of the most interesting texts and novel exam syllabuses (and assessment structures) have stemmed not from publishers, or from the official examination boards, but from teachers' cooperatives - in which teachers who perceive a need work together to produce the necessary materials, to devise the associated structures, and to lobby for the opportunity to put their ideas into practice.

Foremost among such goups in the last 30 years has been the School Mathematics Project (SMP) - which originated in the early 1960s. SMP is a charitable trust, which uses the income generated by earlier ventures to support groups of teachers who see a genuine need and who have the energy to try to meet it. Hence SMP has been a patchwork of many different interests, supporting dozens of different projects - including the most interesting "modern mathematics" series in the 1960s and the first secondary mathematics textbooks which were written for pupils (rather than for the teacher) to read and use in the 1980s.

Another such group of teachers is Mathematics in Education and Industry (MEI) , which has devised interesting exam syllabuses, and associated textbooks, in the spirit of its name.

The increasing pressures on teachers, and the stricter bureaucracy associated with central control of the curriculum and assessment mean that the scope for this kind of innovation is much less than it was.

(b) Competitions : Despite the increasing pressures on teachers and schools, the years since 1988 have seen a dramatic increase in participation in voluntary national competitions aimed at the top 35% of secondary pupils. In 1987 there were essentially two national events for 17-18 year olds involving perhaps 7000 pupils in a small number of schools. Adapting the experience of Australia and Canada, a whole pyramid of events (3 popular multiple-choice papers, 5 harder olympiad papers, and two residential schools covering the 7 secondary years) has been established which now involves nearly 400 000 pupils each year from 60% of the schools in the UK.

(c) Extra-curricular provision : As the previous item illustrates, over the last 20 years extra-curricular provision has depended on the initiative of non-governmental organisations. One such organisation is the Royal Institution, which has coordinated around 40 regional sequences of Saturday morning classes for interested pupils aged 13-15.

In a similar spirit the London Mathematical Society (LMS) appoints and funds four or five lecturers each year who are available to schools to give advertised mathematical talks to pupils from groups of schools. The LMS also runs an annual pair of popular lectures for the general public given by research mathematicians who are renowned for their ability to explain mathematical ideas. The lectures are held each year in three British cities (always including London - where they attract an audience of around 4-500). The lectures are videoed and the videos are made available at a subsidised price.

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